Quasi 4–8 Subdivision Surfaces

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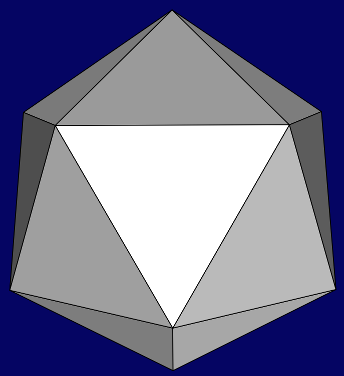
Outline

- Subdivision Surfaces: *Motivation and Background*
- 4–8 Meshes: *Definition and Properties*
- Refinement Operators: *Principles and Methods*
- Quasi 4–8 Meshes: *Subdivision Scheme and Algorithm*
- Results: *Analysis and Examples*

Definition

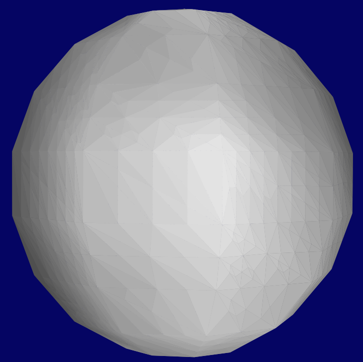
A **Subdivision Surface**, p, is the limit surface generated by a Subdivision Scheme, S, applied to a control polygon, p^0.

$$p = \lim_{n \to \infty} S^n p^0$$



p^0

$$\xrightarrow{S^\infty}$$



p

- Generalization of Spline Surfaces

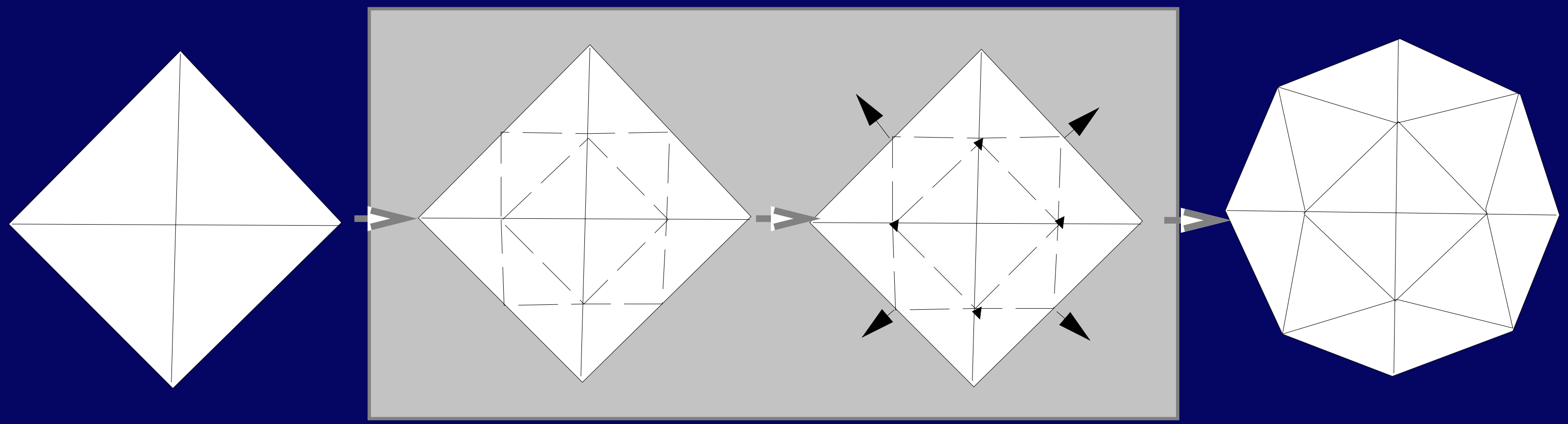
Motivation

- Advantages
  - Control Meshes with Arbitrary Topology
  - Global Smoothness and Local Features
  - Continuous Models from Discrete Representations
  - Simple and Efficient Algorithms
  - Natural Multiresolution Structure
- Disadvantages
  - May not have Closed Form Expression

Subdivision Schemes

- Refinement:
  - *Changes Mesh Topology to increase density*

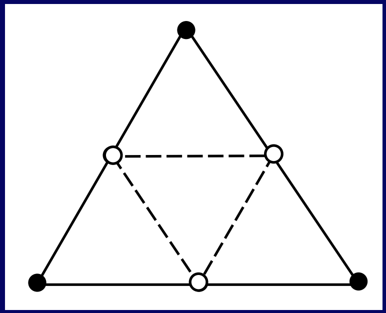
- Smoothing:
  - *Changes Mesh Geometry to increase regularity*



$p^i \rightarrow$ refinement + smoothing $\rightarrow p^{i+1}$

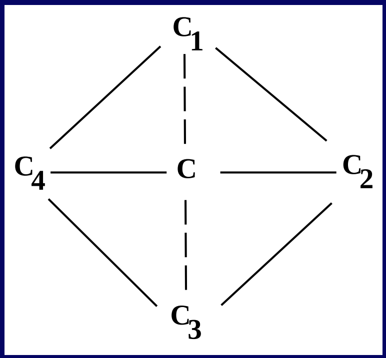
Subdivision Operators

- Refinement Operator, R
  - Subdivision Template
    (○ new vertex, ● old vertex)



- Smoothing Operator, F
  - Filter Mask (Stencil)
    $((c_i)_{i \in N(p)})$

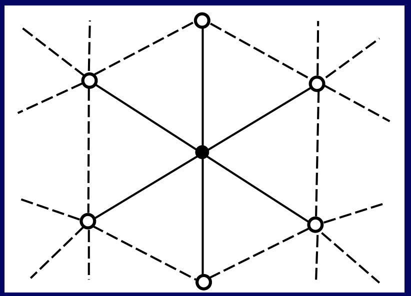
$$p^{j+1} = \sum_{i \in N(p)} c_i p_i^j$$



* Subdivision Operators depend on discrete vertex neighborhoods, $N(p)$

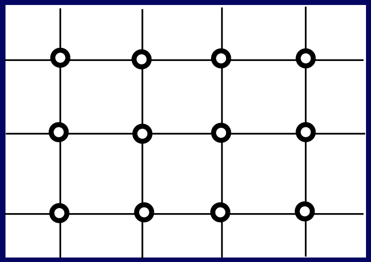
Mesh Structure

- Discrete Local Topology

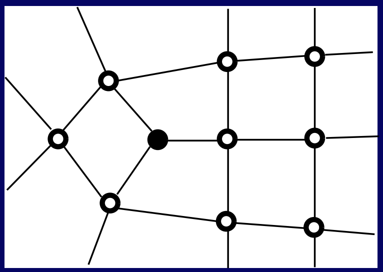


  - 1-Neighborhood of a Vertex: $N_1(v)$
  - Valence (degree) of a vertex: $\deg(v)$

- Classification



  - Regular Meshes (ordinary vertices)

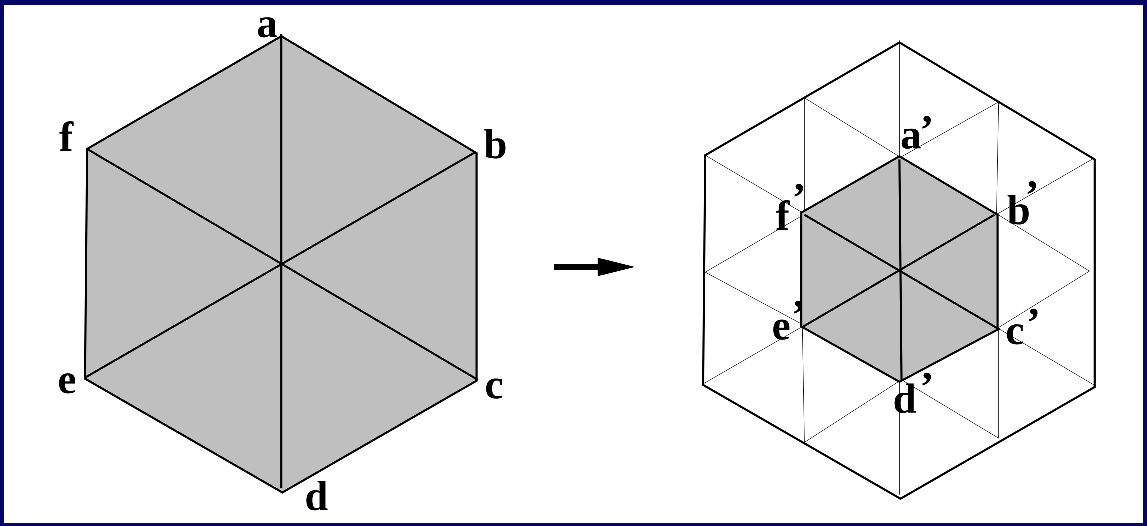


  - Semi-regular Meshes (isolated extraordinary vertices)

Analysis of Subdivision Schemes

Order of Continuity, C^k, of the Limit Surface

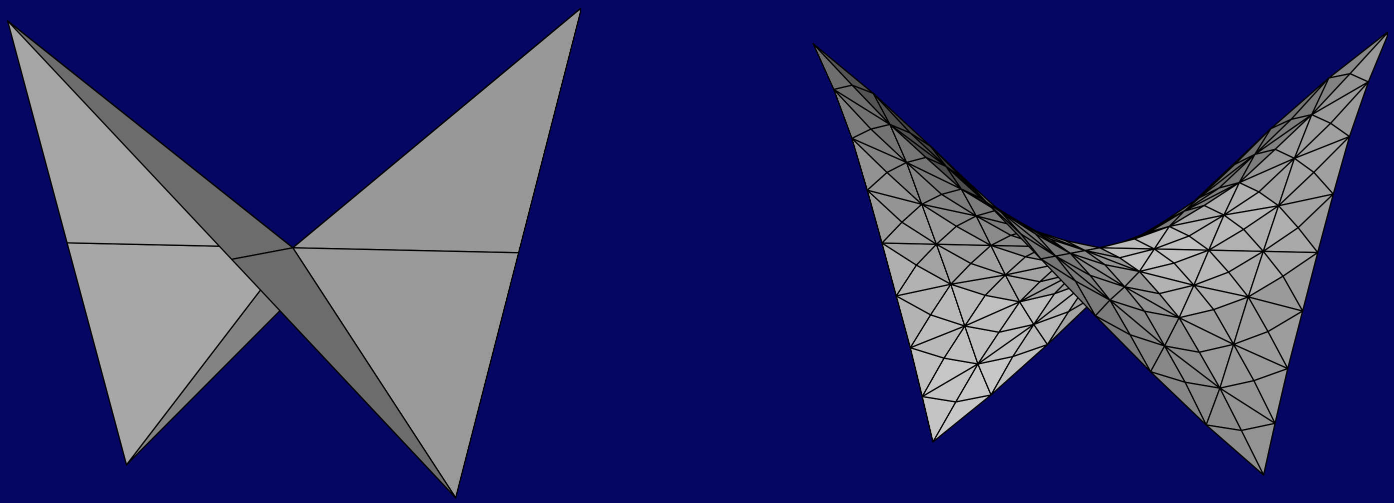
- Convergence Properties: Study *Invariant Neighborhoods*



- Stationary Subdivision: Subdivision Matrix S

Quasi 4–8 Subdivision Surfaces

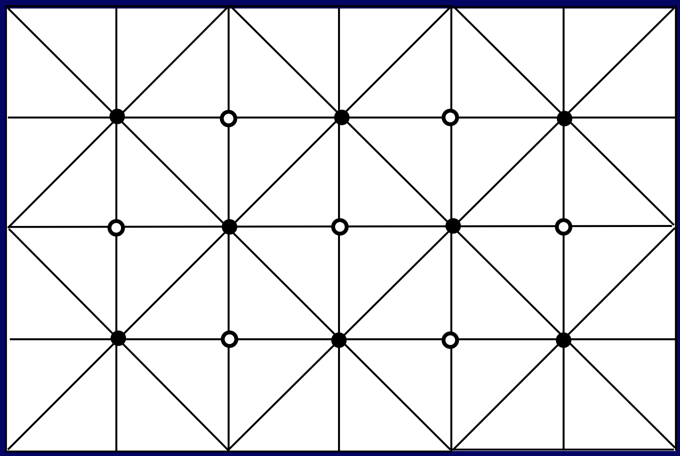
New Subdivision Scheme



- Based on Semi-regular 4-directional Meshes
- Generates Surfaces of class C^1

4–8 Meshes

- Regular 4–directional mesh
  - Generated by the set of vectors: $\{e_1, e_2, e_1 + e_2, e_1 - e_2\}$

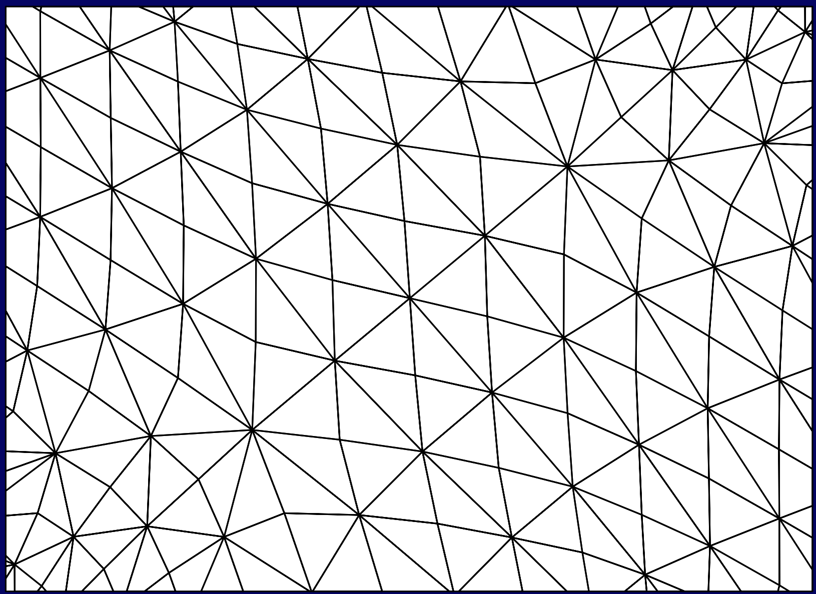


* Regular vertices: valence 4 and 8

* Symmetry Properties: Leaves Tiling $[4.8^2]$

Quasi 4–8 Meshes

- Semi-regular 4–direction Mesh

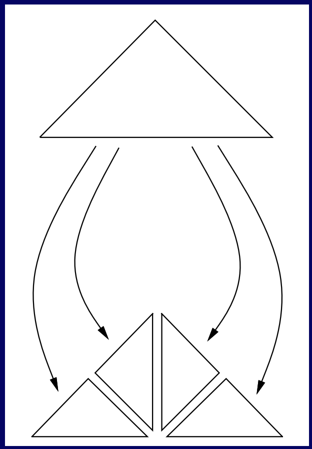


* *Most Properties of 4–8 Meshes*

* *Generated by Refinement*

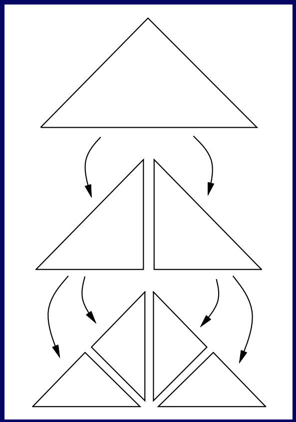
Refinement of 4–8 Meshes

- Normal Refinement (quaternary)



  1. Split edges in 2
  2. Subdivide face in 4

- Interleaved Refinement (recursive binary)



  1. Split main edge
  2. Subdivide face in 2
  3. Refine subfaces

Quasi 4–8 Subdivision Scheme

- Overview

  1.a Find Maximal Independent Set of 2-Face Clusters

  1.b Perform Binary Subdivision on Faces

  2. Apply Smoothing Filter to Vertices

- Algorithm

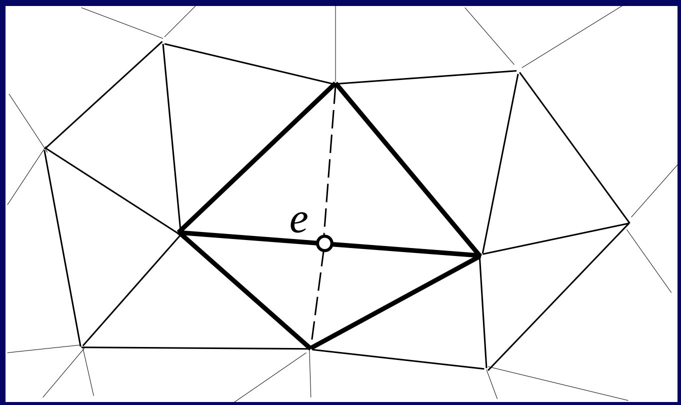
quasi_4–8_subdivision (K)

  quasi_4–8_refinement (K)

  quasi_4–8_smooting (K)

Refinement Operator

- Select Clusters based on Edge Length



– Binary Subdivision Template

– Internal Edge Bisection

Refinement Algorithm

quasi_4–8_refinement (K)

sort_edges (E)

store $e \in E$ in priority queue Q

while $Q \neq \emptyset$ **do**

get e from Q

if e not marked **then**

split (e)

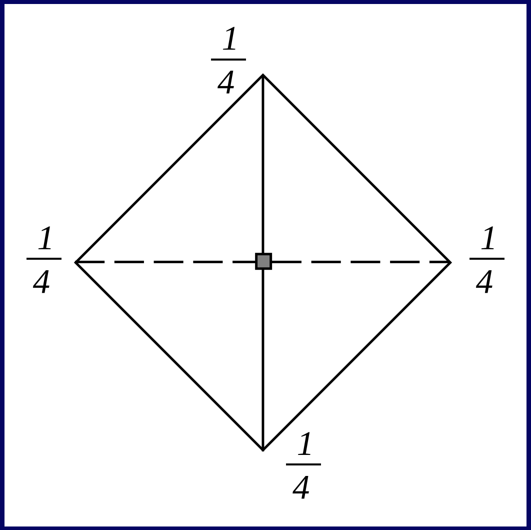
mark_cluster (e)

for all $f \in F$ **do**

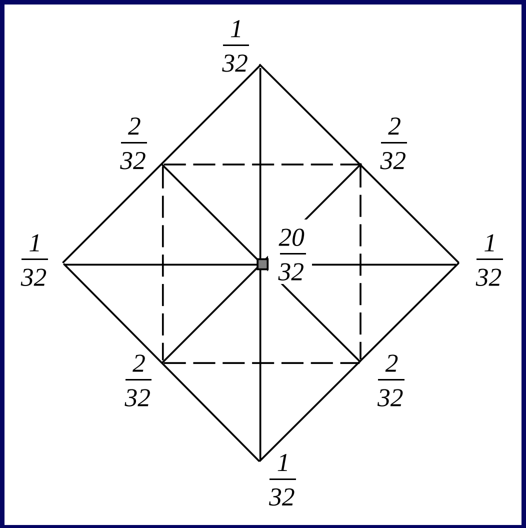
subdiv (f)

Smoothing Operator

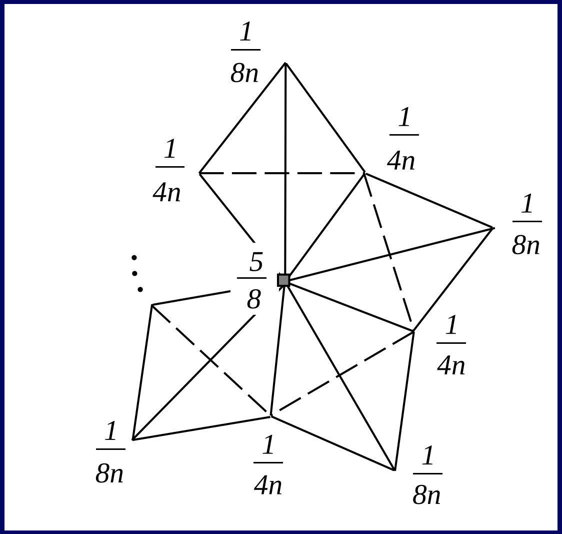
- Smoothing Templates



New Vertices



Ordinary Vertices



Extraordinary Vertices

Smoothing Algorithm

quasi_4–8_smoothing (K)

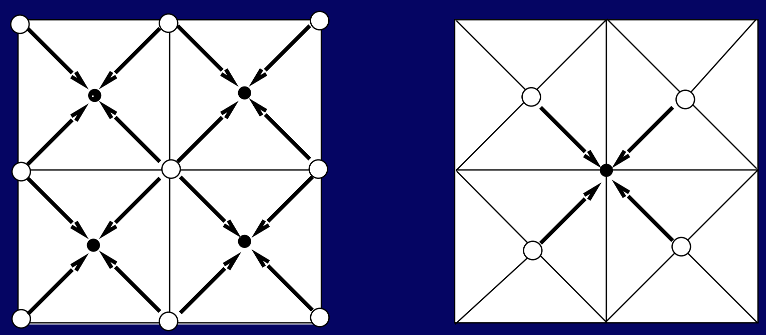
for $v_i' \in V'$ **do**

$$p^{l+1}(v_i') = \frac{1}{4} \sum_{v_j \in N_1(v_i')} p^l(v_j)$$

for $v_i \in V$ **do**

$$p^{l+1}(v_i) = \frac{1}{2} p^l(v_i) + \frac{1}{2n} \sum_{v_j' \in N_1(v_i) \cap V'} p^{l+1}(v_j')$$

- Cascade Convolution



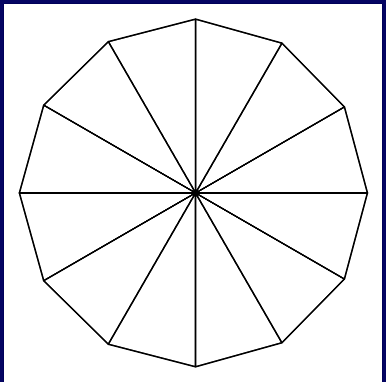
Convergence of the Scheme

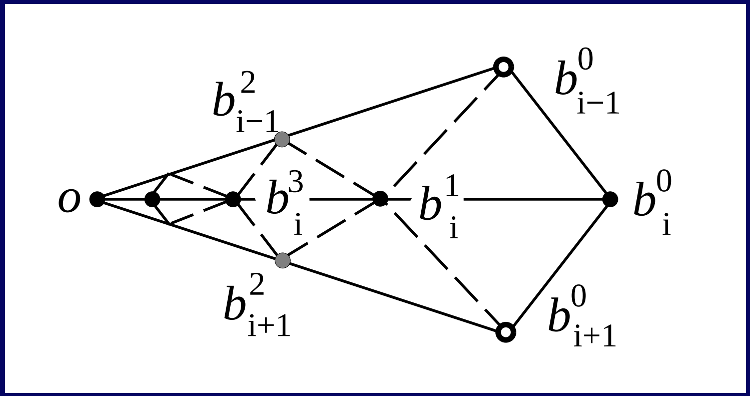
- Quasi-Stationarity
  - Geometric reasoning based on planar case
- C_1 Continuity
  - Eigenanalysis of the Subdivision Matrix
  - Investigation of Characteristic Map

Quasi–Stationarity

- Planar Case

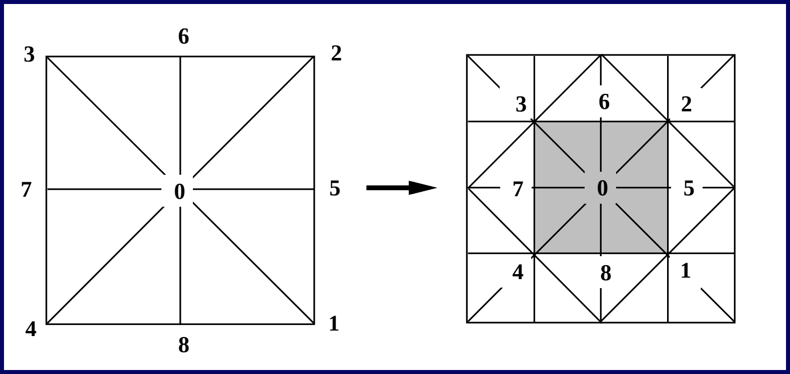
1. newly inserted vertices have valence 4.
2. old vertices of valence 4 change to valence 8.
3. old vertices of valence 8 do not change.
4. vertices of any other valence change little.





C^1 Continuity

- Invariant Neighborhood of Regular Vertex $v \in V_8$



- Subdivision Matrix S_8

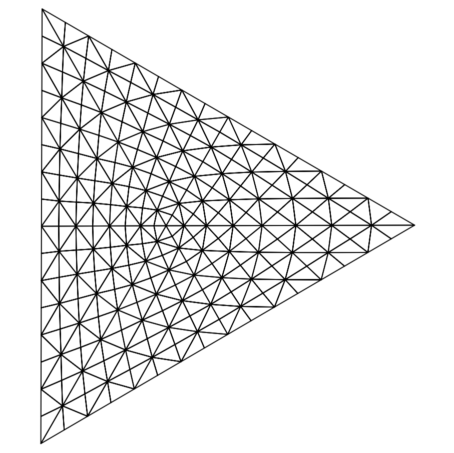
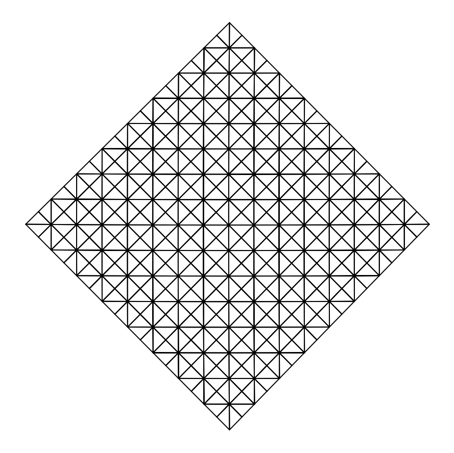
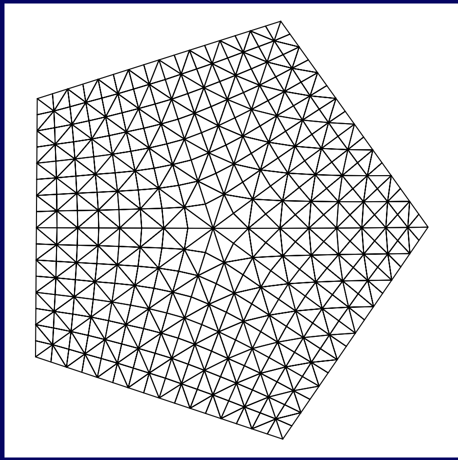
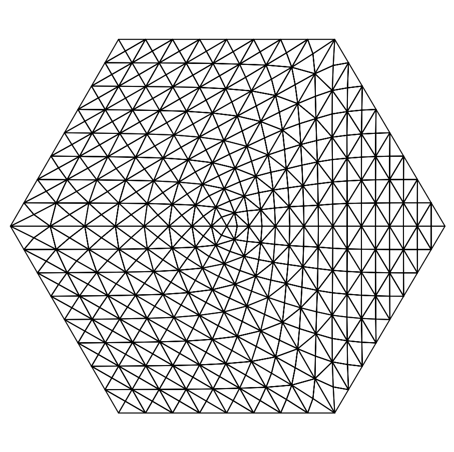
$$S_8 = \frac{1}{32}\begin{pmatrix} 20 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 2 \\ 8 & 8 & 0 & 0 & 0 & 8 & 0 & 0 & 8 \\ 8 & 0 & 8 & 0 & 0 & 8 & 8 & 0 & 0 \\ 8 & 0 & 0 & 8 & 0 & 0 & 8 & 8 & 0 \\ 8 & 0 & 0 & 0 & 8 & 0 & 0 & 8 & 8 \\ 12 & 2 & 2 & 0 & 0 & 12 & 2 & 0 & 2 \\ 12 & 0 & 2 & 2 & 0 & 2 & 12 & 2 & 0 \\ 12 & 0 & 0 & 2 & 2 & 0 & 2 & 12 & 2 \\ 12 & 2 & 0 & 0 & 2 & 2 & 0 & 2 & 12 \end{pmatrix}, \qquad \lambda_i = \{1, \frac{1}{2}, \frac{1}{2}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}\}$$

1. $\lambda_0 = 1 > \lambda_1 \geq \lambda_2 > |\lambda_3|$;

2. eigenvector corresponding to λ_0 is $(1, 1, 1, 1, 1, 1, 1, 1)$;

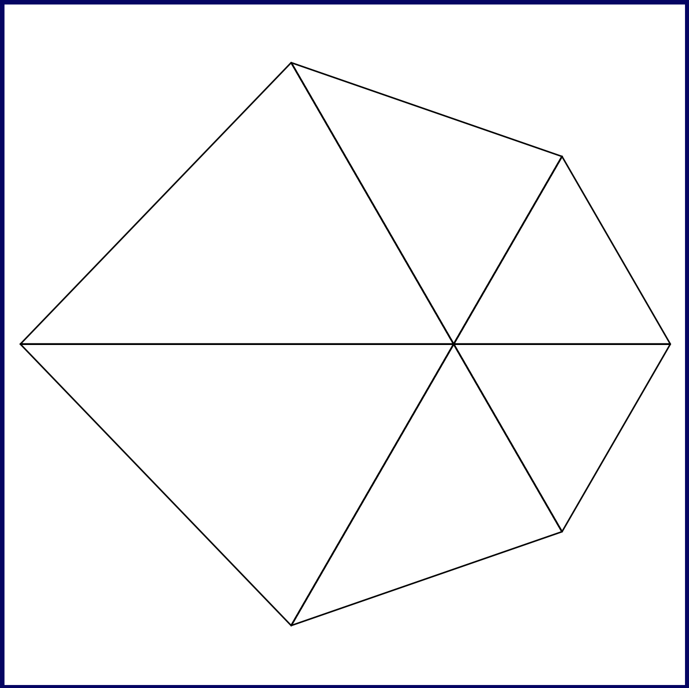
Subdivision of N–Regular Planar Polygons

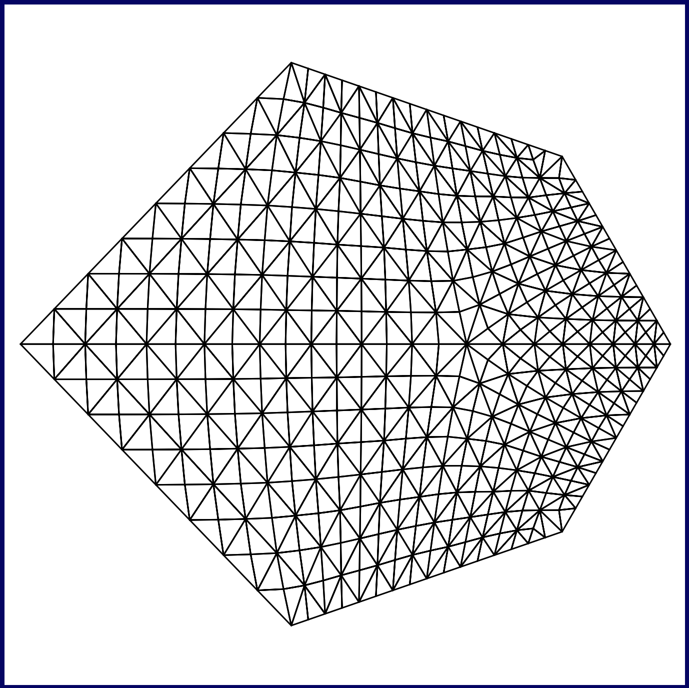
- Intuition for Characteristic Map Induced by S_k

Subdivision of Warped Hexagon

- Adaptive Refinement

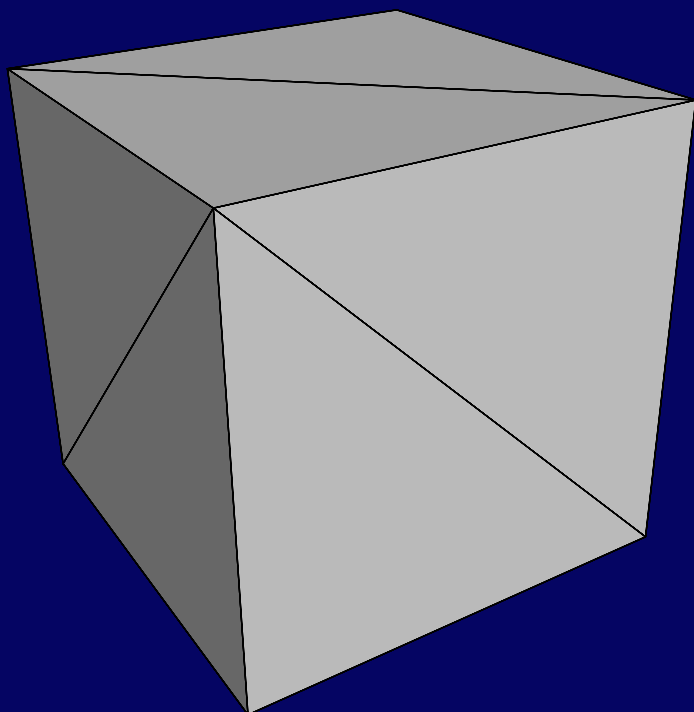


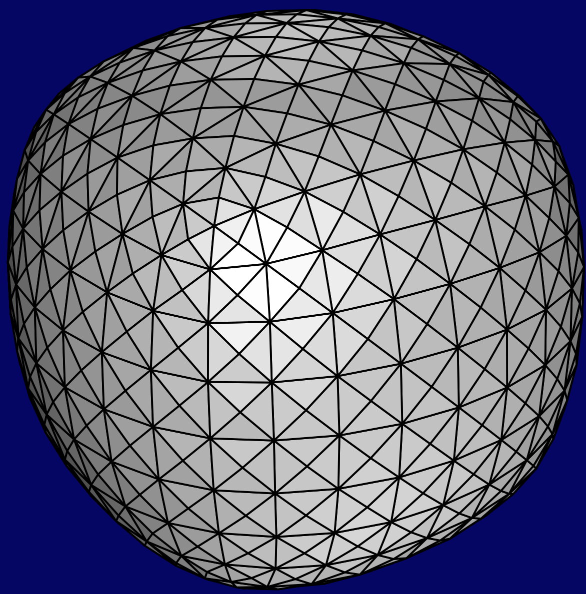


Examples

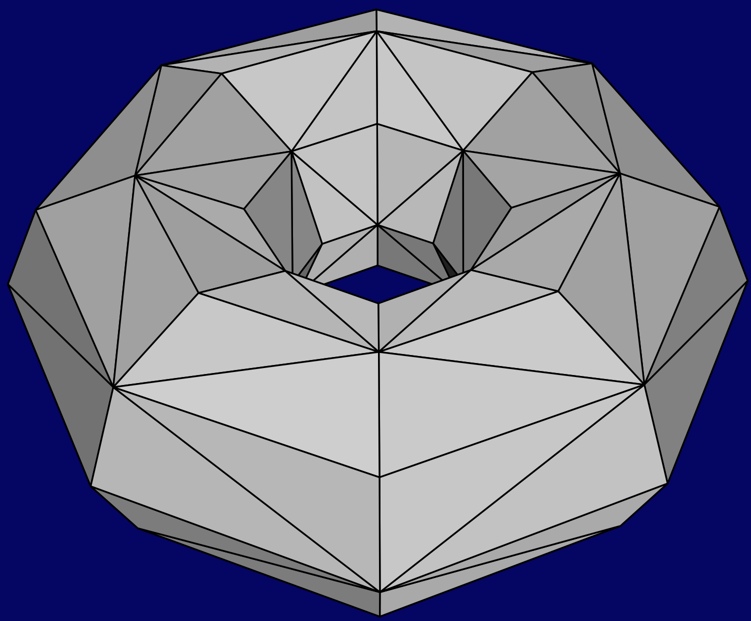
- Simple Shapes
  - Cube: genus 0 surface
  - Torus: genus 1 surface
- Feature Control
  - Boundary and Creases
- Complex Objects
  - Bunny
  - Cow

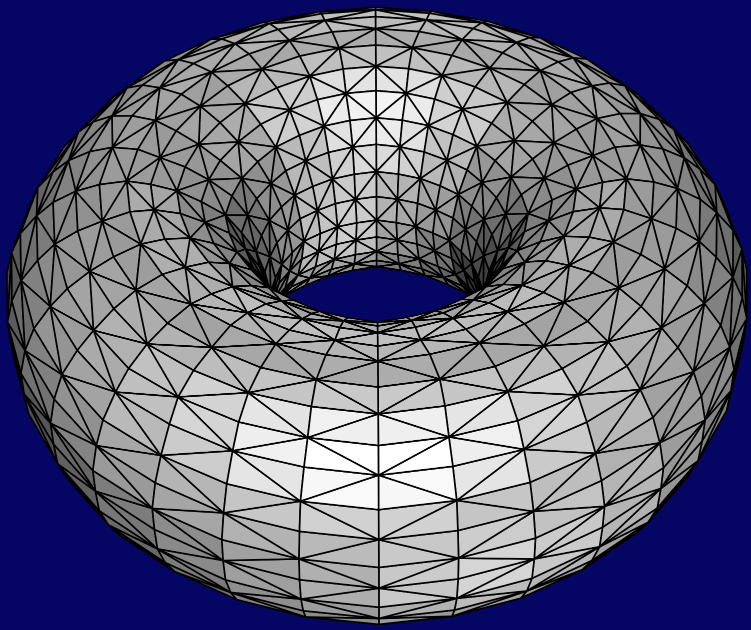
Cube





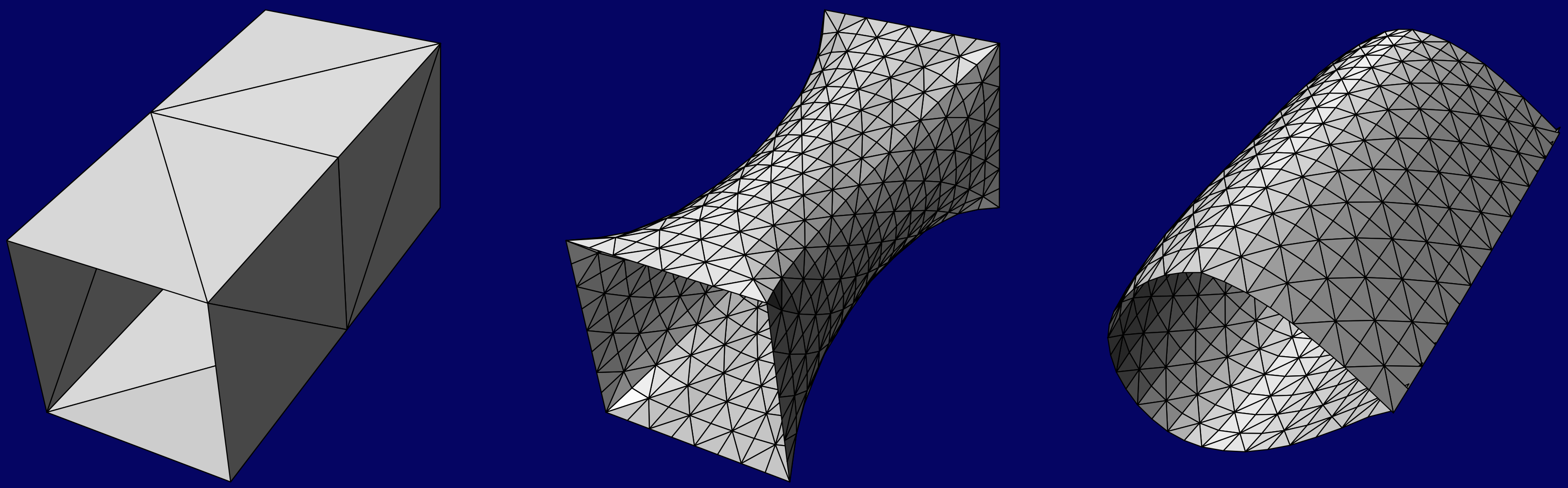
Torus



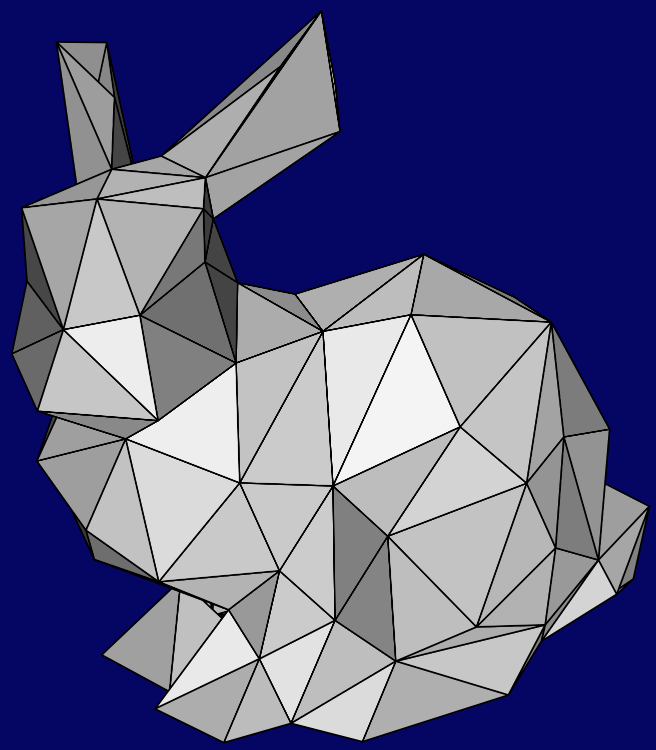


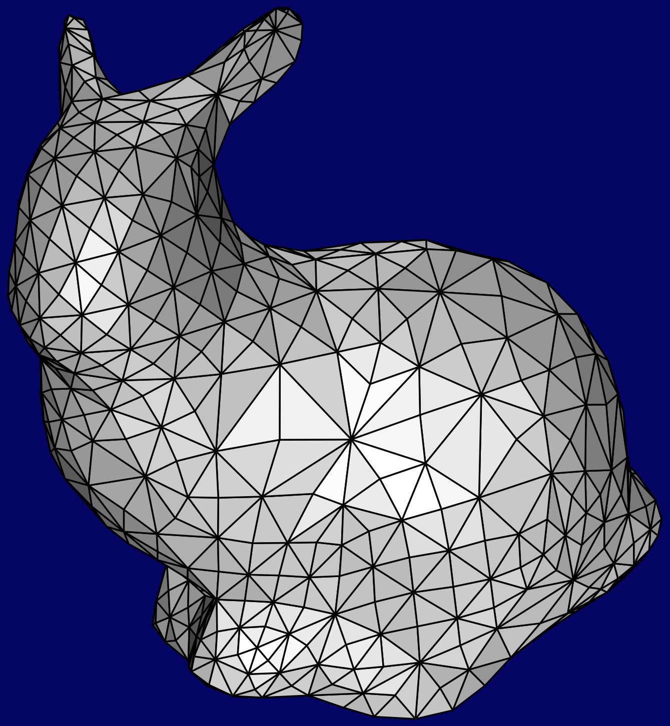
Cylinder

- Feature Control: Tagged Edges

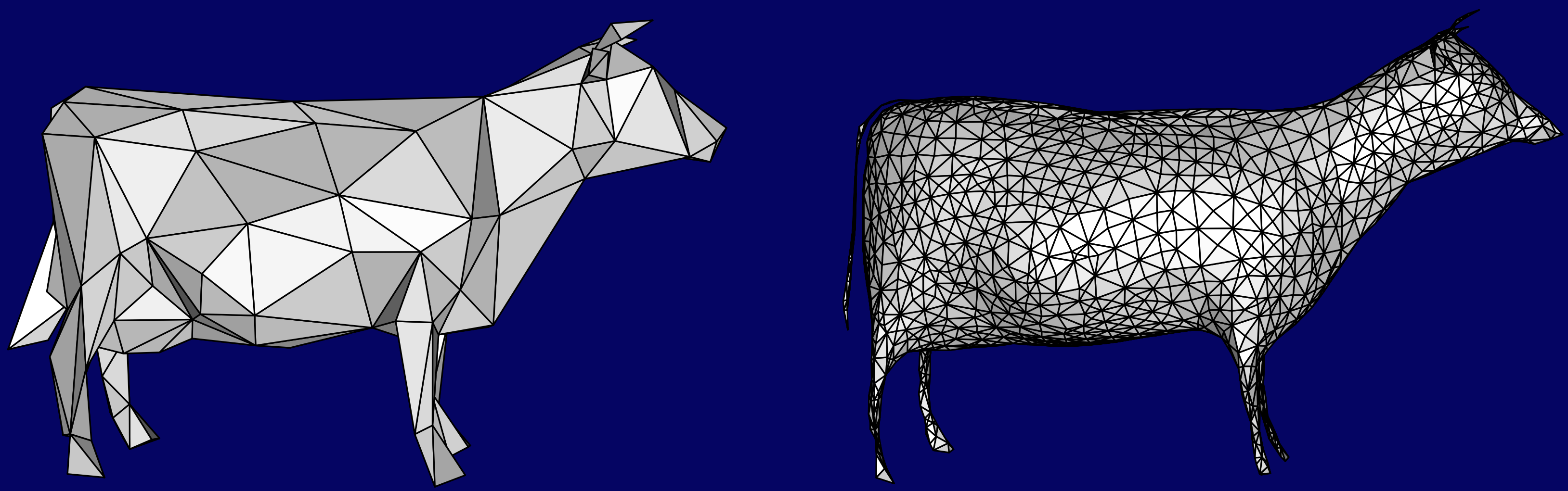


Bunny





Cow



Conclusions

- New Subdivision Scheme
  - Well Coupled Operations
  - Adapted Meshes
  - Simple and Efficient
- New Concepts
  - Geometric-Sensitive Refinement
  - Quasi Stationary Subdivision