4–8 Subdivision Surfaces

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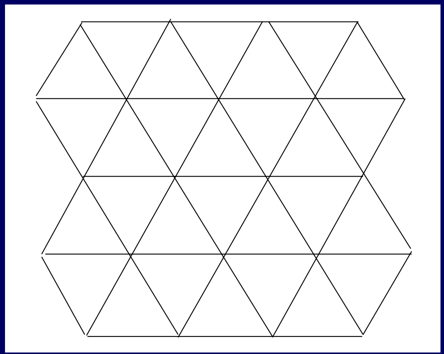
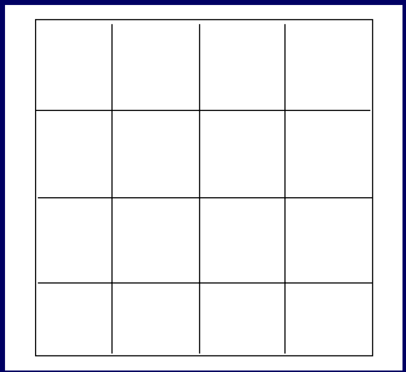
Outline

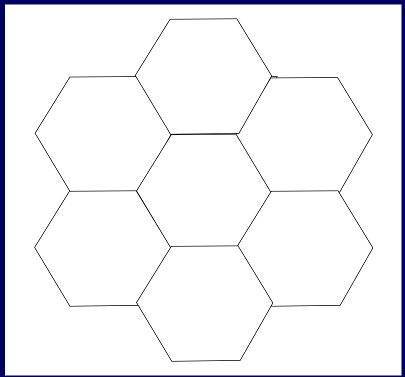
- Semi-Regular 4–8 Refinement
  - Adaptive Tilings
  - Refinement Procedure
- Decomposing Subdivision Rules into Binary 4–8 Steps
  - Four Directional Box Spline
  - Catmull-Clark Subdivision Surface

Regular Meshes

- Regular Tilings of the Plane

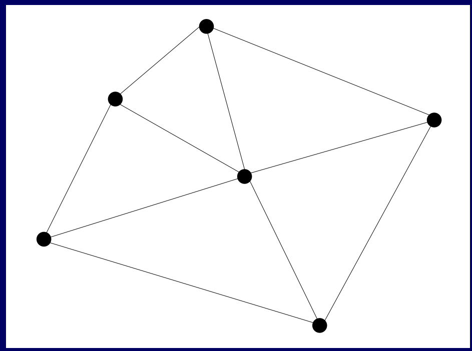

Triangular

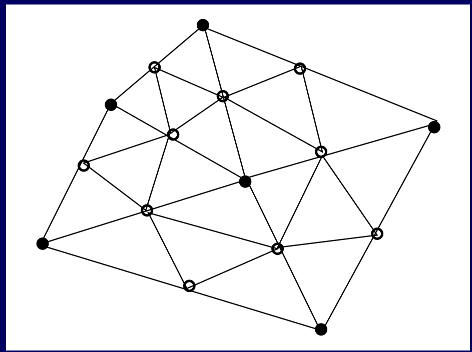

Quadrilateral


Hexagonal

* Main Property for Subdivision Surfaces
  - *Refinability*, (define a multiresolution)

Refinable Combinatorial 2D Manifolds



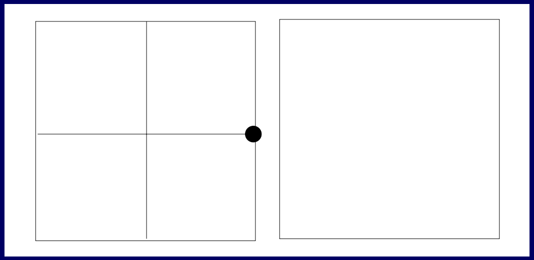


- Semi-Regular Refinement
  - Irregular Control Mesh
  - Regular Refinement Rule
- Types of Vertices
  - ○ Ordinary Vertices
  - ● Extraordinary Vertices

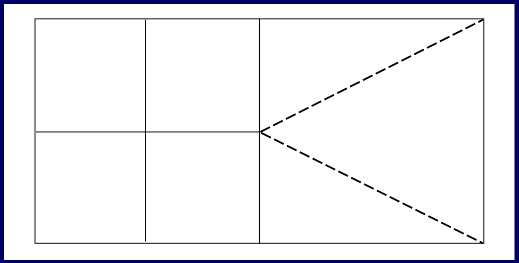
* Subdivision Surfaces of Arbitrary Topology

Uniform Refinement

- Regular Tilings force Global Splitting (i.e. *cannot be adaptive*)



- Ad-Hoc Solutions: (*adopted by all current subdivision schemes*)

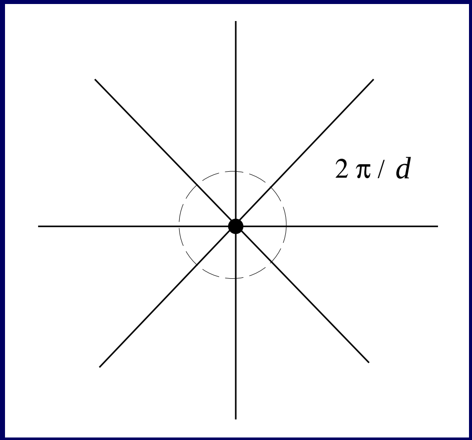


- Search for a Better Solution:
  non-uniform refinement, (regular and semi-regular)

Laves Tilings

- Definition: (properties)
  - Monohedral Tilings (*tiles are congruent*)
  - Regular Vertices (*angles are* $2\pi/d$)

* *Superset of Regular Tilings*

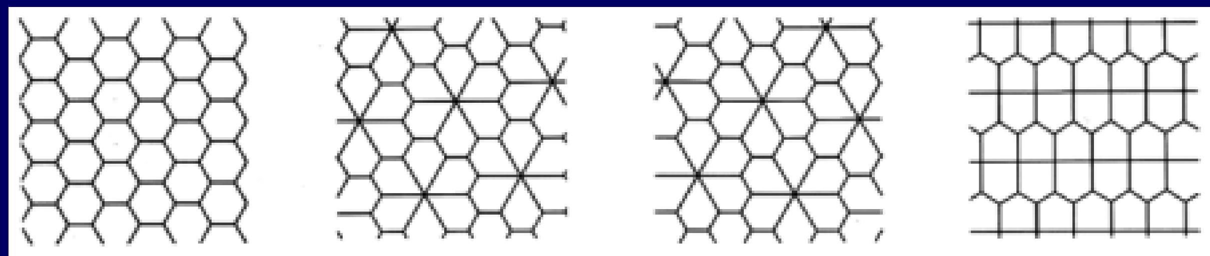


- Comparison with Regular Tilings
  - Same Shape but not Regular
  - More than one Type of Regular Vertex

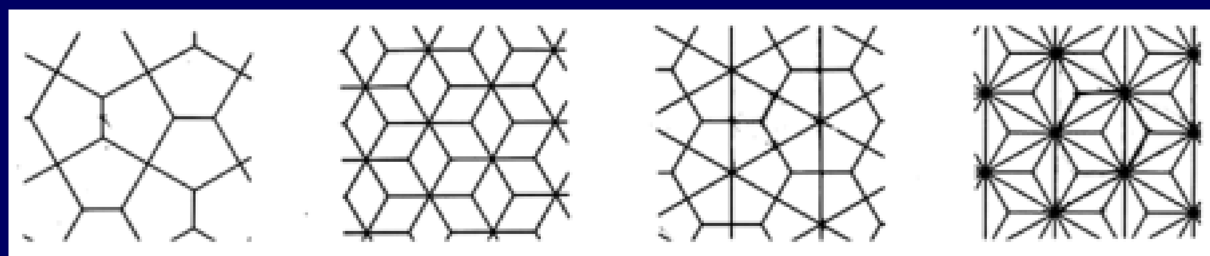
$\Rightarrow$ *Can be used in a Subdivision Scheme*

Classification of Laves Tilings

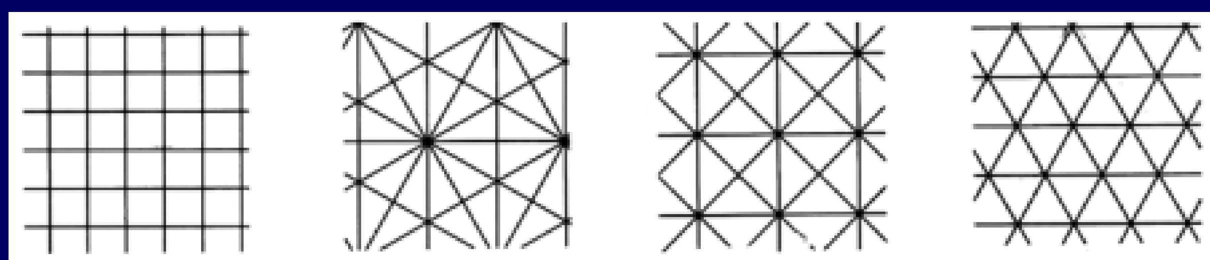
11 Tilings



$[3^6]$ $[3^4.6]$ $[3^4.6]$ $[3^3.4^2]$



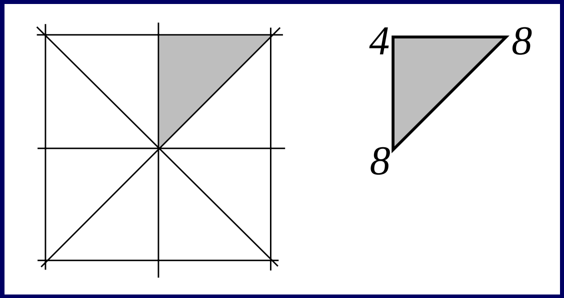
$[3^2.4.3.4]$ $[3.6.3.6]$ $[3.4.6.4]$ $[3.12^2]$



$[4^4]$ $[4.6.12]$ $[4.8^2]$ $[6^3]$

Notation: *named after the degree of vertices of Prototile*

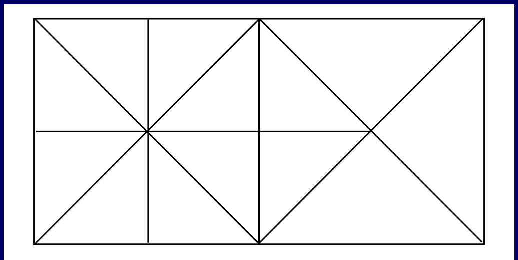
The $[4.8^2]$ Laves Tilings



– *Prototile:* Right Triangle

– *Regular Valences:* 4 and 8

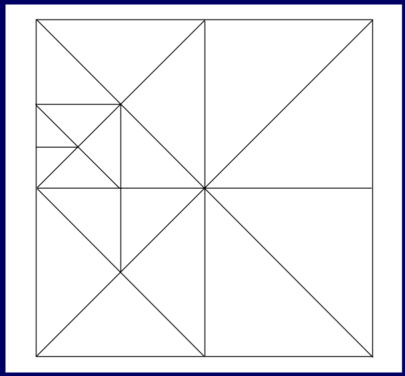
* *Simplest tiling that supports regular, non-uniform refinement*



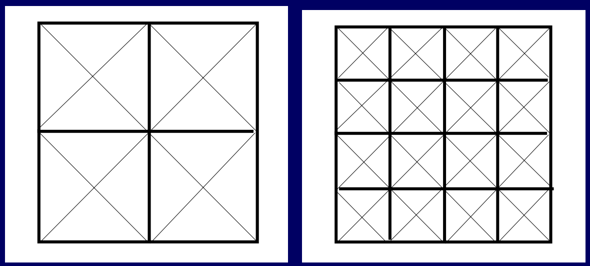
OBS: Triangulated Quadrangulation

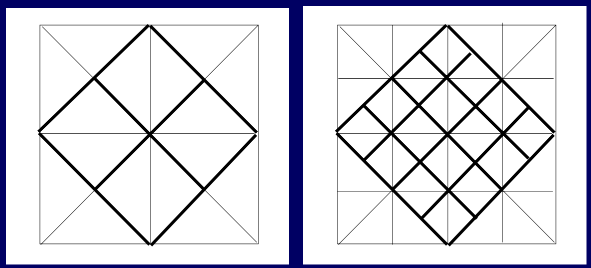
Mesh Adaptivity

- Restricted Structure: *Smooth Transitions*



- Two Interleaved Quadtrees: *Variable Resolution*

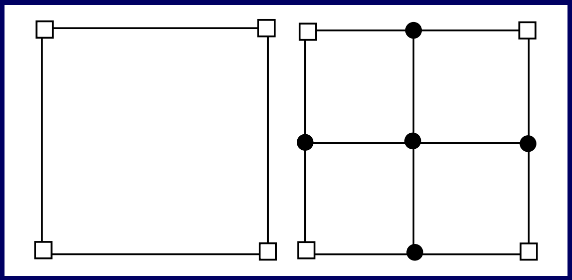




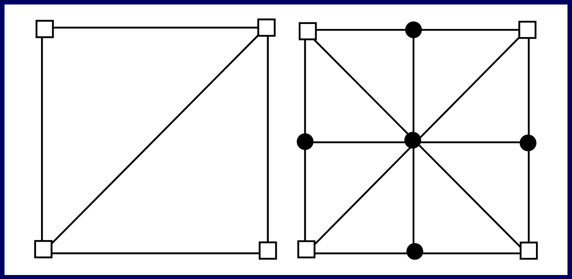
Splitting Quadrilateral Blocks

- Primal Split

Quad mesh



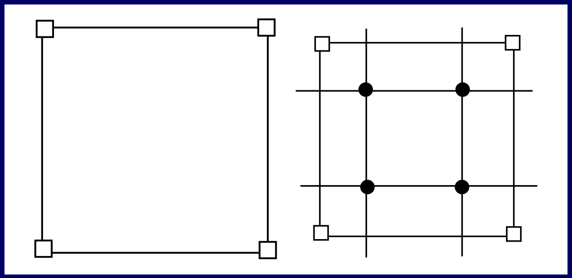
4–8 mesh



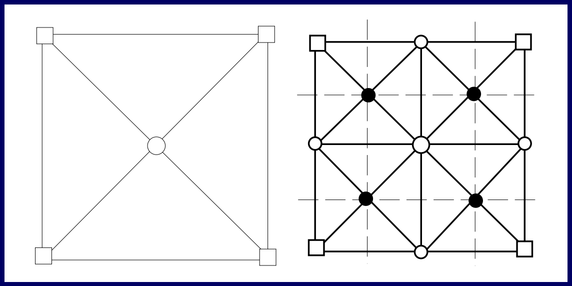
before after

- Dual Split

Quad mesh



4–8 mesh



before after

Subdivision using $[4.8^2]$ Tilings

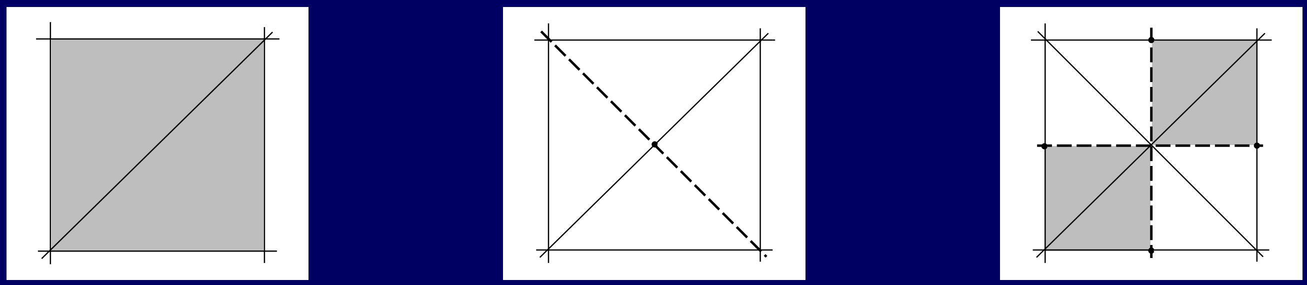
- Advantages
  - Combine:
    . Regular and Non-Uniform Refinement
  - Integrate:
    . Triangular and Quadrilateral Meshes
  - Unify:
    . Primal and Dual Split

* *Need to Develop a Refinement Method*

  - Regular
  - Semi-Regular

Regular Refinement

- Interleaved Binary Subdivision (double step)



i $\quad$ $i+1$ $\quad$ $i+2$

* *Acts on Two-Face Clusters*

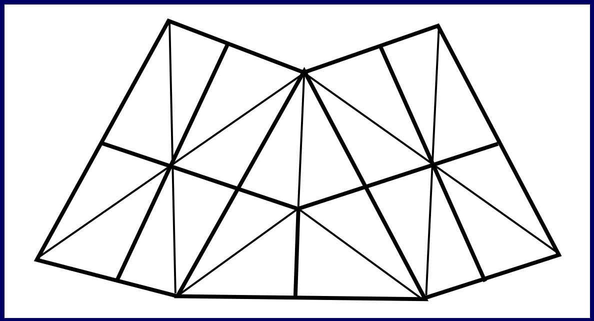
- Binary Subdivision Procedure

1. Split 8–8 Edges

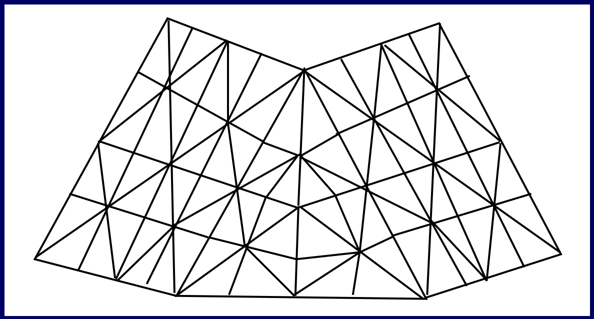
2. Subdivide Faces into Two

Semi-Regular Refinement

1. Initialization: (Triangulated Quadrangulation)

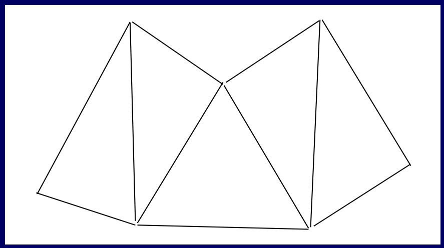


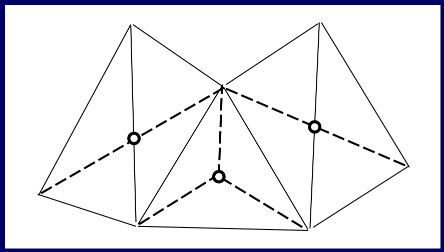
2. Regular refinement: (Interleaved Binary Subdivision)



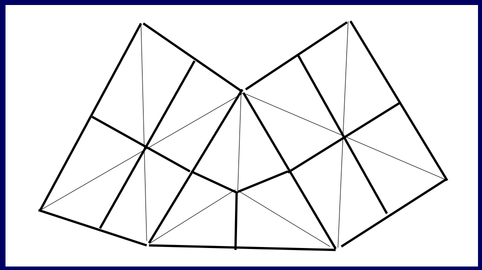
Construction of the Base Mesh

1. Compute Independent Set of:
   two-tace clusters + isolated triangles





2. Hybrid Binary Subdivision:

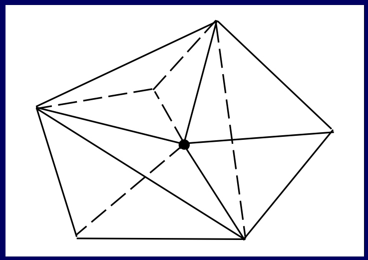


3. One Step of Regular 4–8 Refinement:

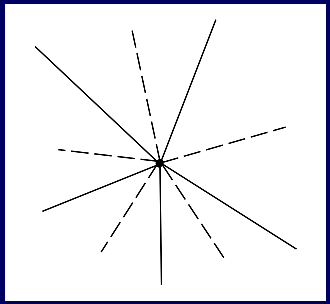
* OBS: *Equivalent to Catmull-Clark First Split*

Analysis of the Method

- *Only the Initialization step changes valences of initial vertices*
- *4-8 Refinement creates valence 4 vertices that change to valence 8*
- As a consequence:

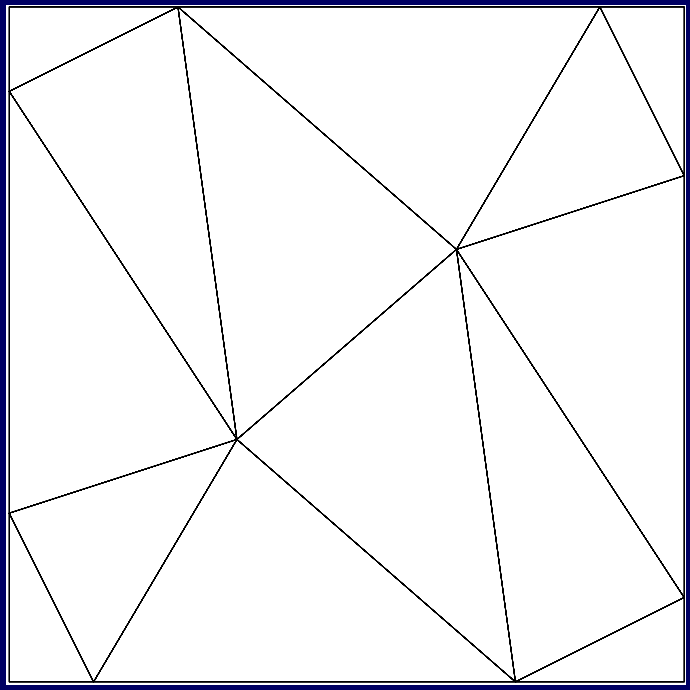


  - All vertices will have <u>even valence</u>
  - Theoretical worst case: $n \mapsto 2n$
  - Verified in practice:
    - $n = 3$ — new valence 6;
    - $4 \leq n \leq 8$ — new valence ≈ 8;
    - $n > 8$, odd — new valence $n+1$;
    - $n > 8$, even — new valence n.

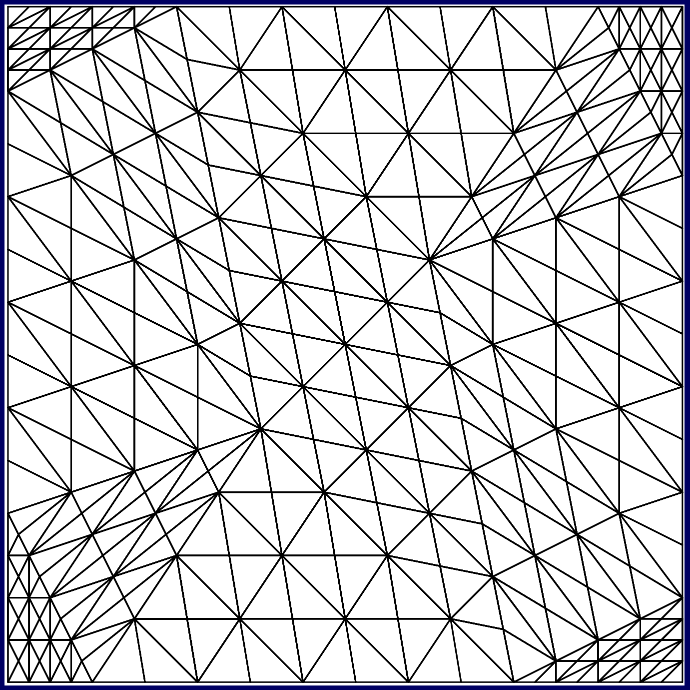


Examples

- Planar Mesh



base mesh



2 levels of refinement

4–8 Subdivision Schemes

- Subdivision Scheme: (2 Operators)
  - 4–8 Refinement
  - 4–8 Smoothing

* *How to implement the smoothing operator?*

- Examples
  - C^1 Four Directional Box Splines:
    . four directional grid, dual refinement
  - Catmull-Clark Subdivision Surface:
    . quadrilateral grid, primal refinement

Computation of Smoothing Rules

- Defined over Underlying Quadrangulation:
  . Quadrilateral Blocks – (*primal and dual*)

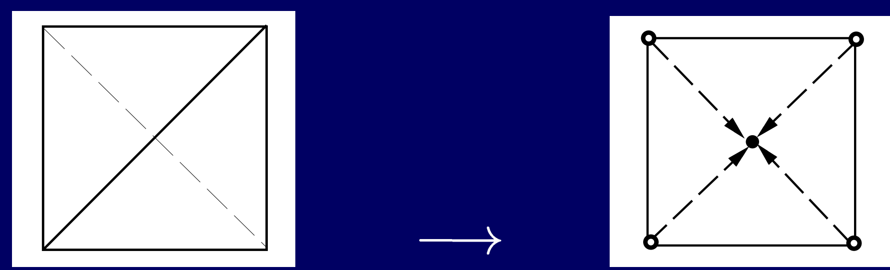
- Exploit Properties of Refinement Operator:
  . Double-step refinement $\Rightarrow$ *decomposable*

- Efficient Implementation:
  . Factorization of smoothing rules – (*intermediate results*)

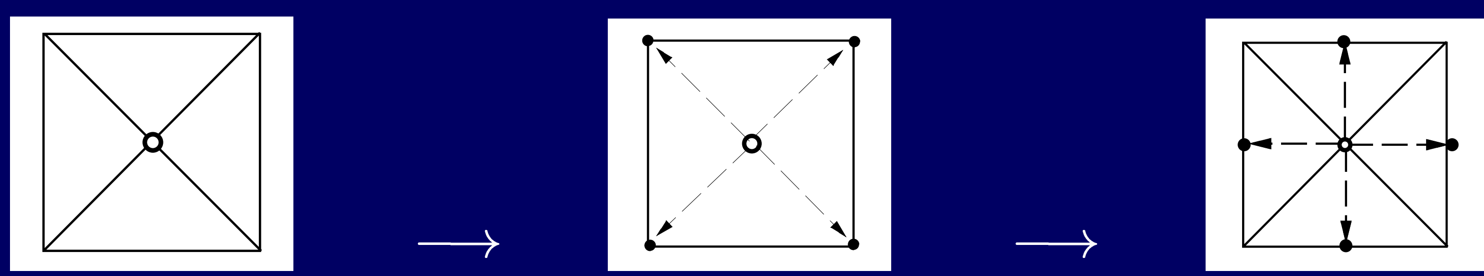
Factorization of Smoothing Rules

- Basic Strategy:
  1 - Divide Vertices in Two Classes: new and old vertices
  2 - Decompose Computation in Two Phases: collect and distrib

  – Collect:



  – Distribute:



. intra step inter step

C^1 Four Directional Box Spline

- Zwart-Powell basis: *piecewise quadratic box spline*

- Direction Vectors:

$$D = \begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{pmatrix}$$

- Generating Function:

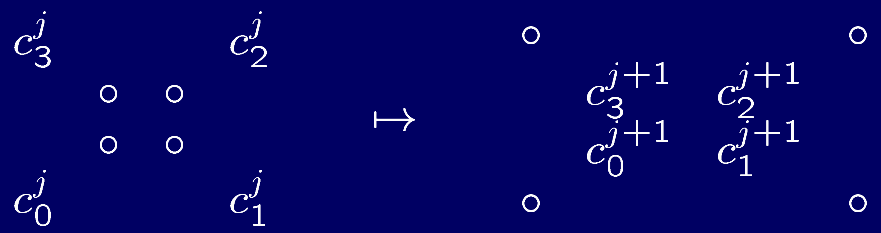
$$S(z_1, z_2) = \frac{1}{4}(1 + z_1)(1 + z_2)(1 + z_1 z_2)(1 + z_1/z_2)$$

- Coefficients Subdivision Formula:

$$\begin{matrix} & 1/4 & 1/4 & \\ 1/4 & 1/2 & 1/2 & 1/4 \\ 1/4 & 1/2 & 1/2 & 1/4 \\ & 1/4 & 1/4 & \end{matrix}$$

Subdivision Operator

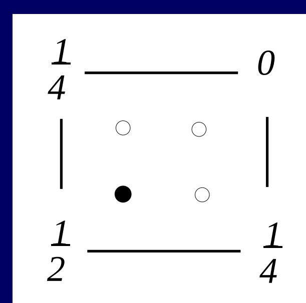
- Dual Subdivision Scheme



- Smoothing Rule

$$c_0^{j+1} = \frac{1}{2}c_0^j + \frac{1}{4}c_1^j + \frac{1}{4}c_3^j$$

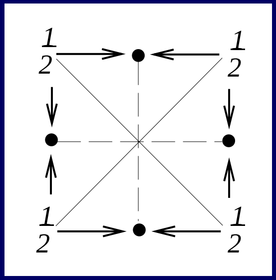
- Smoothing Mask



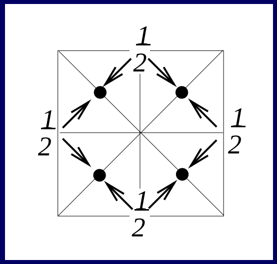
Decomposition of the Operator

- Factorization:

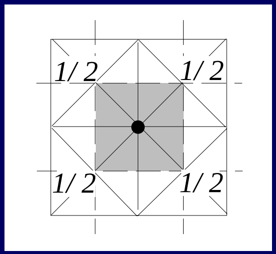
step i

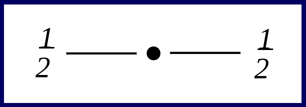


step $i+1$



- Updating Center of Quad-Blocks

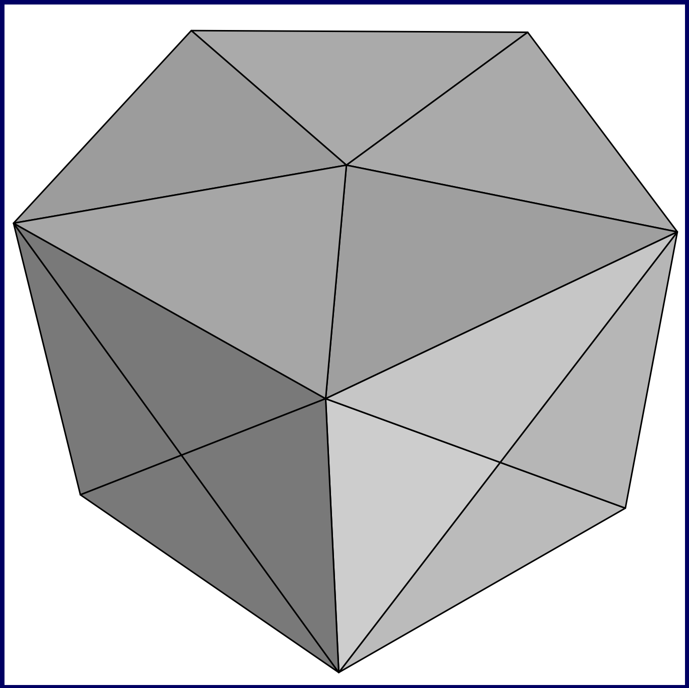


- Mask: 

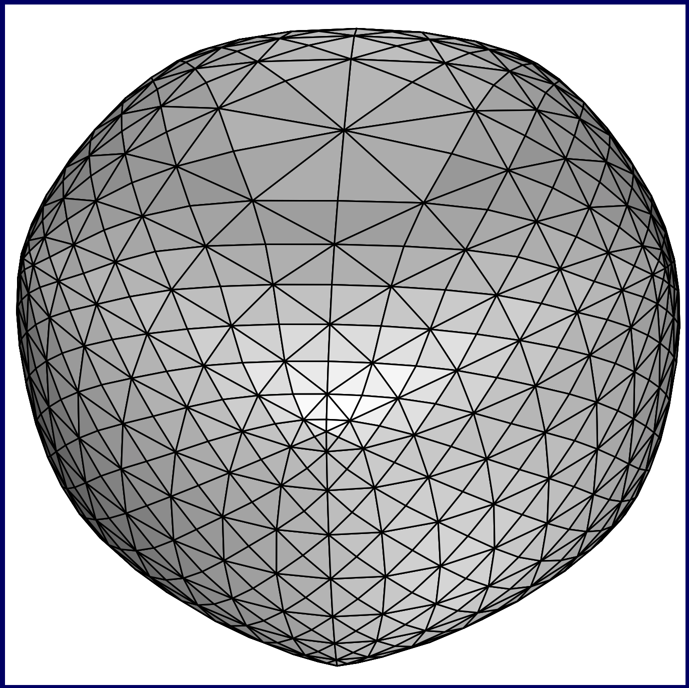
- Remark: *convolution along grid lines*

Example

- Extruded Pentagon



base mesh



smoothed (2 levels)

Catmull-Clark Subdivision Surface

- Bicubic Tensor Product B-Spline

- Equivalent to Box Spline with Direction Vectors:

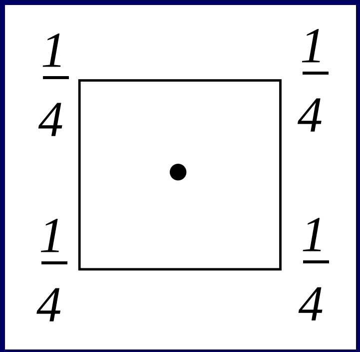
$$D = \begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

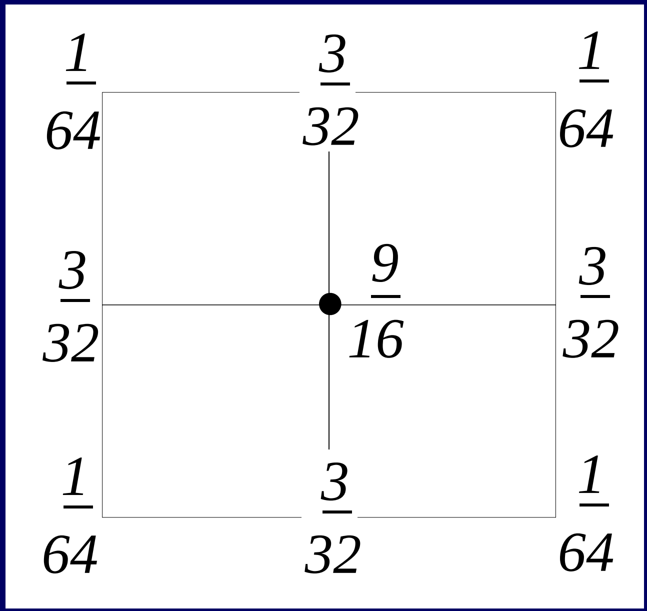
- Generating Function:

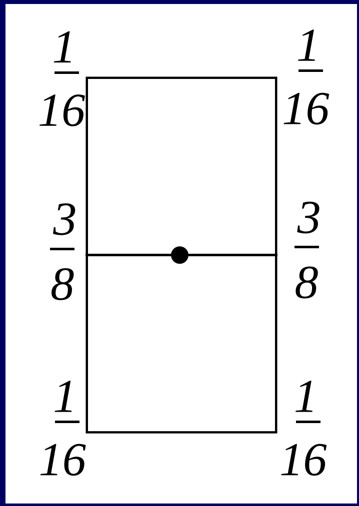
$$S(z_1, z_2) = \frac{1}{64}(1 + z_1)^4(1 + z_2)^4$$

Subdivision Operator

- Rules for Ordinary Vertices:







face

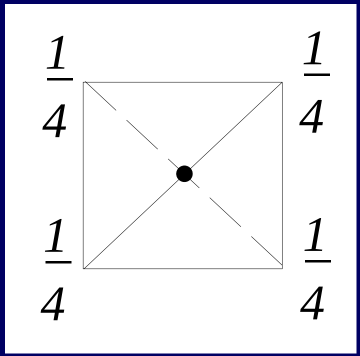
corner

edge

Decomposition of the Operator

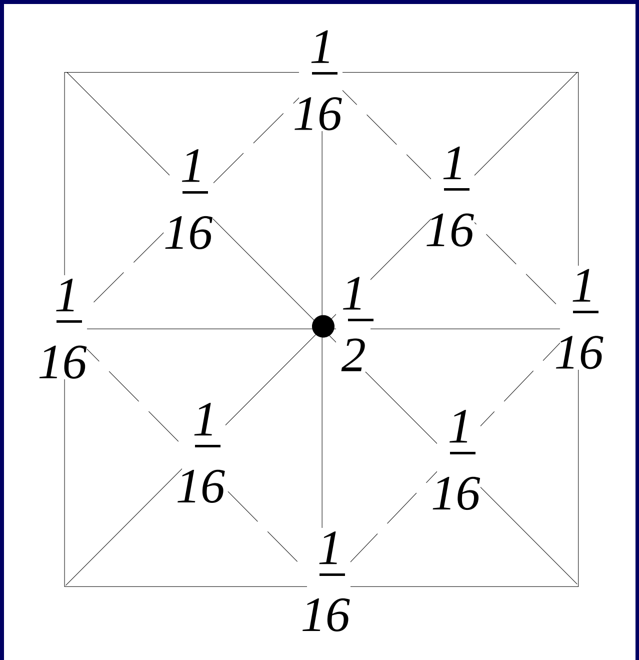
- Factorization:

step i



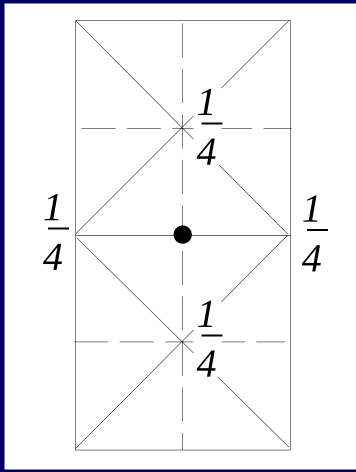
face

step $i+1$



corner

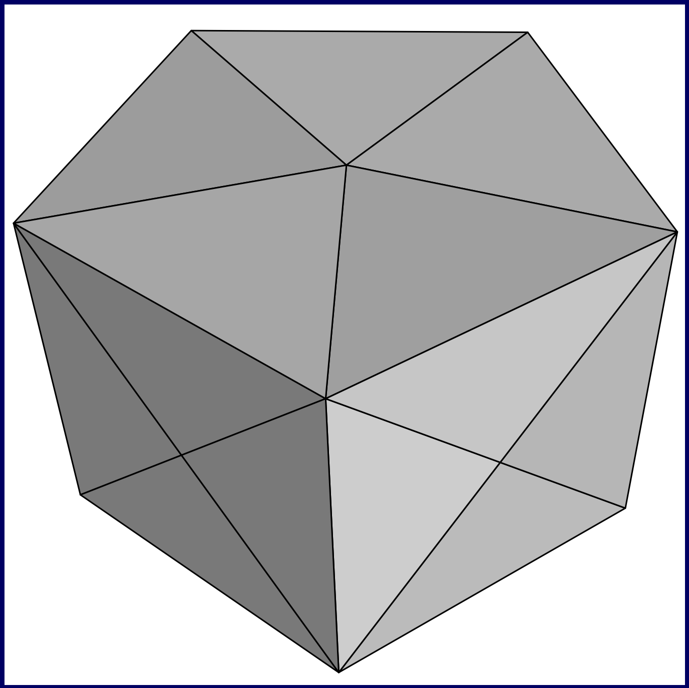
step $i+1$



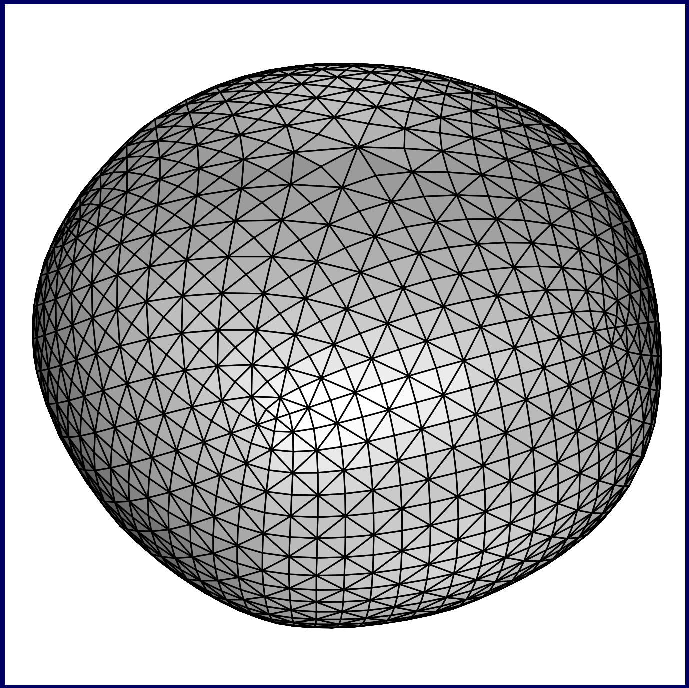
edge

Example

- Extruded Pentagon



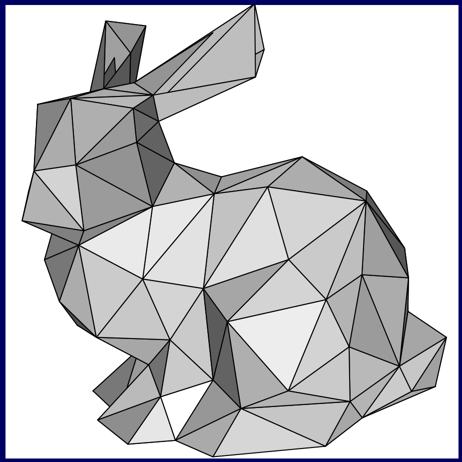
base mesh

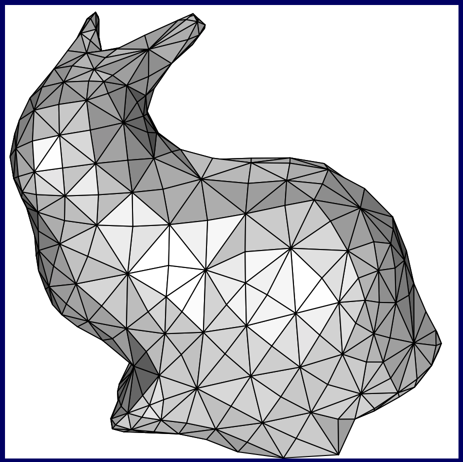


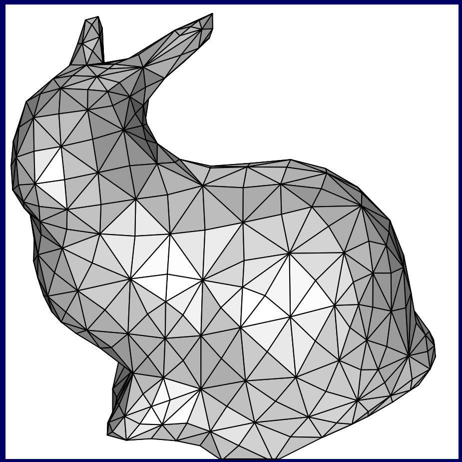
smoothed (2 levels)

Comparison Between the Two Schemes

- Stanford Bunny



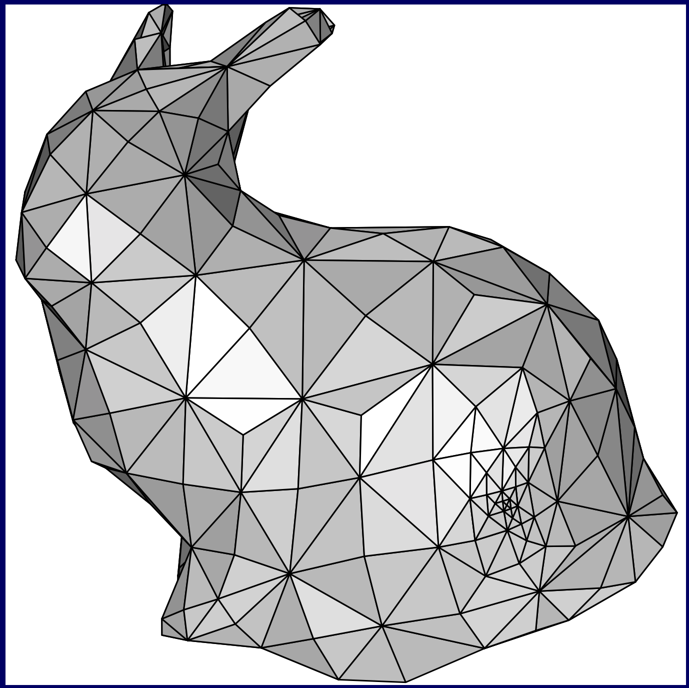




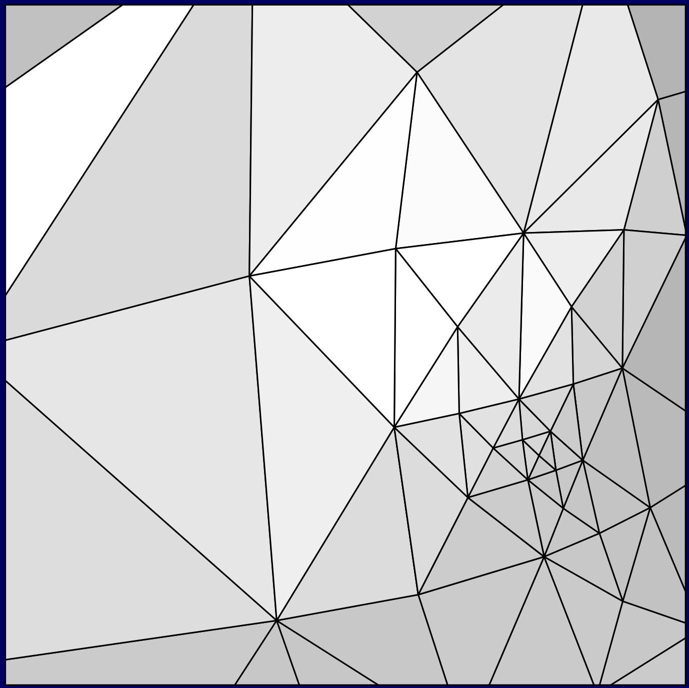
base mesh Zwart-Powell Catmull-Clark

Adaptation

- Point Location



adapted mesh



transition region

Conclusions

- New Scheme: 4–8 Subdivision Surfaces
  - Semi-regular Refinement
  - Factorization of Smoothing Rules
- Future Work
  - Study Convergence at Extraordinary Vertices
  - Investigate Relation with *Lifting Scheme*
  - Build Associated Wavelets