Image Restoration Using Non-Decimated Wavelet Transform and Row-Action Projections

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Abstract. This project tries to apply two different methodologies for image restoration. A previous step, named wavelet denoising is applied before the deblurring algorithm, which is developed using POCS techniques.

1. Introduction

Image restoration techniques are normally used to increase the definition of images obtained by some kind of sensors, like CCD. Atmospheric turbulence, optical aberrations and sensor movement affect the images obtained with a detector, reducing its sharpness.

The image restoration problem has been widely studied due its numerous pratical applications as well as its theoretical interest. Thus the blurred image can be expressed by a system of linear equations [1] given by

$$g = Hf + n \tag{1}$$

where *g*, *f* and *n* are the lexicographic row-ordered vectors of the discretized versions of the blurred and original image and the additive random noise respectively. *H* is the degradation matrix composed of the PSF.

In that way, the image restoration methodology can be divided in two other problems: image denoising and image deblurring. In this work we adopted two different approaches for each problem described above.

2. Image Denoising

A one-dimensional (1-D), discrete wavelet transform (DWT) is a linear operation *W*, mapping an input array *y* with length *N* onto an array of wavelet coefficients *w*

$$w = Wy\,. \tag{2}$$

It proceeds as a repeated (recursive) filterbank with a lowpass filter $\tilde{h}$ with elements $\tilde{h}_i$ and a highpass filter $\tilde{g}$ with elements $\tilde{g}_i$. Reconstruction uses both filters with their elements, respectively [2]. For image restoration we need a two-dimensional (2-D) transform, which can be obtained by performing separable transforms, on columns and rows.[i]

2.1 Non-Decimated Wavelet Transform

A modified version of the tradicional wavelet transform DWT is known as Non-Decimated Wavelet Transform (NDWT) or stationary wavelet transform, which has no subsampling step and therefore keeps the same number of coefficients of each level. The reconstruction procedure for this redundant data representation is immediately extensible for cases where the number of data is not a power of two.

2.2 Wavelet Thresholding

The main goal of this first step is to perform the image denoising in the wavelet domain, due to the fact that the DWT divides the signal in two parts: highpass and lowpass frequencies. We know that the additional noise is located on the high frequencies, so the key idea is that the wavelet representation can separate the signal and the noise. The DWT compacts the energy of the signal into a small number of DWT coefficients having large amplitudes, and it spreads the energy of the noise over a large number of DWT coefficients having small amplitudes. Hence, a thresholding operation attenuates noise energy by removing those small coefficients while maintaing signal energy by keeping these large coefficients unchanged [3].

We suppose the input for our algorithm is given by Equation (1). A wavelet transform yields the same situation in terms of wavelet coefficients:

$$w = v + e\,, \tag{3}$$

where the vector $v = W(Hf)$ contains the wavelet coefficients of the blurred data, $e = Wn$ are the noise coefficients and $w = Wg$.

For the wavelet shrinkage we use soft threshold, given by

$$w_\delta = \begin{cases} w-\delta, \; if \; w > \delta \\ w+\delta, \; if \; w < -\delta \\ 0, \; otherwise \end{cases} \tag{4}$$

To obtain the optimal threshold δ, we used Generalized Cross Validation (GCV) [2].

3. Image Deblurring

For the second part, which corresponds to image deblurring, we used the methodology known as Projections onto Convex Sets (POCS). The POCS method can be used to find a common point *f* which satisfies a set of constraints, each of which forms a convex set. This common point *f* lies in the intersection of all the convex sets [4]

$$f \in C = \bigcap_{i=1}^{i=m} C_i \tag{2}$$

where the *i*th closed convex set C_i denotes the *i*th constraint on *f*. The common point can be found by alternatively projecting onto the convex sets C_i via the corresponding projecting operator P_{C_i} as

$$f^{(k+1)} = P_{C_m} P_{C_{m-1}} \ldots P_{C_1} f^{(k)} = P_C f \tag{3}$$

The RAP algorithm, which is considered a subset of POCS, forms a solution to a set of linear equations given by Equation (1), by iterative orthogonal projection onto the hiperplanes especified by each equation.

The RAP update equation is given by

$$f^{(k+1)} = f^k + \lambda \frac{g_p - h_p^T f_k}{\left\| h_p \right\|^2} h_p \tag{4}$$

where λ is the relaxation factor, g_p is the *p*th element of the vector *g* and h_p^T is the *p*th row of the matrix *H*. Therefore the RAP algorithm can be generalized to include constraints in each iteration and can be rewritten as a projection operator of POCS, just replacing $f^{(k+1)}$ and $f^{(k)}$ by $P_{C_R} f$ and $P_C f$ respectively.

4. Preliminary Results

We first applied the wavelet denoising algorithm to a phantom image Lena, obtained by blurring the image with a 2-D gaussian with $\mu = 0$ and $\sigma = 16{,}75$. After we added gaussian noise, with $\mu = 0$ and $\sigma = 5$. Figure 1 displays the (a) original, (b) degraded and (c) denoised images, respectively.

After, we applied the RAP algorithm to the denoised image. Figure 2 shows (a) the restored image and (b) the restored image without denoising step.

To evaluate our algorithms we used the improvement signal to noise ratio (*ISNR*). Using the wavelet denoising step we obtained $ISNR = 1.52$, and without denoising $ISNR = 0.75$.

(a) (b) (c)

Figure 1 (a) Original image, (b) degraded image and (c) denoised image.

(a) (b)

Figure 2 (a) Restored image using wavelet denoising and (b) restored image without wavelet denoising.

5. Conclusion

We concluded that with the wavelet denoising our images were better restored, due the fact that this step tries to eliminate the additive noise.

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