Extrapolation for Signal and Image Restoration

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Abstract

Microscopic images, specially with a nonconfocal microscope, are fundamentally limited because the optical transfer function, the Fourier transform of the point-spread function, is zero over a region of the spatial-frequency domain. Several iterative algorithms are being developed for the restoration and superresolution of diffraction-limited imagery data through the use of different mathematical techniques. Here we present the Gerchberg-Papoulis algorithm and test it in a 1-d signal and also in a real microscopic image.

1. Introduction

Like the human eye, most instruments cannot discern fine details. Microscopic images are often used on medical and biological research and are severely affected by blurring. Through modern restoring methods, scientists have access to information that would otherwise remain unavailable. Since the formation of an image alters the recorded information content from that of the original object, there has been much interest directed to processing images so as to more closely match the original object. Credible methods appear in recent years for the reconstruction of spatial frequencies of the object that are beyond the diffraction limit [1]. Processes that achieve the restoration of frequencies beyond the image or signal passband are often called *spectrum extrapolation* or *superresolution* algorithms.

A classical algorithm that appears to solve this problem is the Gerchberg-Papoulis algorithm [2] [3]. This algorithm still remains as a model for comparision with new techniques.

In this work we will describe this algorithm and test it on a one-dimensional signal to extrapolate in time domain. We also present some blurred images from a nonconfocal microscope that we desire to restore.

2. Microscopy Images

The image formation system can be modeled as follows:

$$\mathbf{g} = \mathbf{H}\mathbf{f} \qquad (1)$$

where $\mathbf{g}$ is the observed image, $\mathbf{f}$ is the ideal image and $\mathbf{H}$ is the point spread function (PSF). It works as a low-pass filter limiting the frequencies that pass through the system and blurring images (Figure 1).

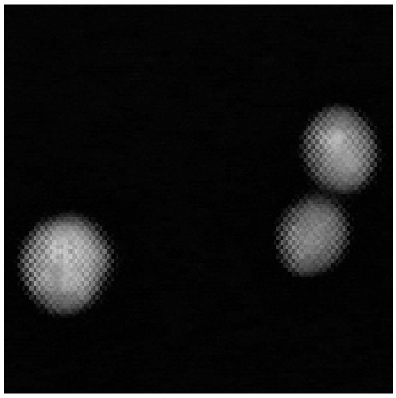

Figure 1. Microscopy image of a cell culture

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3. Extrapolation

Fourier optics demonstrates that there exists a cut-off spatial frequency, which is directly determined by the shape and size of the limiting pupil in the optical system [4]. This distortion of the spatial frequency components is governed by the optical transfer function (OTF), the Fourier transform (FT) of the point spread function (PSF) of an imaging system. Beyond the diffraction limit cut-off frequency no spatial frequency information about the object is available, i.e., PSF is zero beyond the diffraction limit, in order that is not just a matter of an inverse problem to restore the lost frequencies.

3.1. Gerchberg-Papoulis Algorithm

Papoulis [2] and Gerchberg [3] formulated a simple technique to implement extrapolation as follows:

Assume we are given a band-limited object $g(x)$ and have some knowledge about the bandwidth Ω of $g(x)$. The Gerchberg-Papoulis (G-P) algorithm iteratively imposes the requirements that the signal is band-limited and matches the known portion of the signal. It consists generally of the following steps:

1. Initiate iteration $N = 0$ and $f^N(x) = g(x)$.
2. Pass $f^N(x)$ through a low-pass filter within the known bandwidth Ω.
3. Use the FT of the filtered signal and set to zero outside interval of the passband.
4. Perform the Inverse Fourier Transform (IFT) of the clipped signal.
5. Impose the known portion of the observed signal, $g(x)$, to the processed signal.
6. The new signal is no longer bandlimited. Go to step 2 and iterate until desired convergence.

4. Experiments

Here we test the G-P algorithm on a clipped version of the function: $f(x) = \sin(2\pi x)/2\pi x$. (Figure 2-a). The algorithm was applied using 10 iterations. This example is commonly used to explain the theory of G-P algorithm restoring the signal in time domain [5].

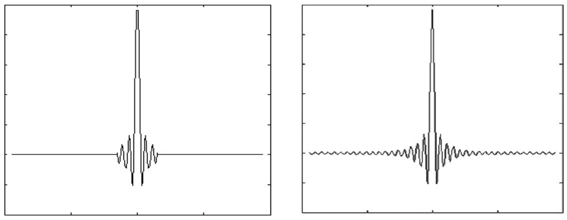
Another experiment used a real nonconfocal microscopy image (power spectrum of this image is shown in Figure 3-a), performing extrapolation with use of 1000 iterations. Extrapolation of spectrum was obtained using a low-pass filter in frequency-domain outside the central region of power spectrum [6]. After IFT, pixels with values outside a [0 255] interval were replaced by values of the original image pixels.

5. Results

Using the 1-d signal the algorithm was able to restore some frequencies. The restored signal is shown in Figure 2-b. Some extrapolation on the microscopy image was obtained. Restored frequencies appear in the power spectrum of the restored image (Figure 3-b).

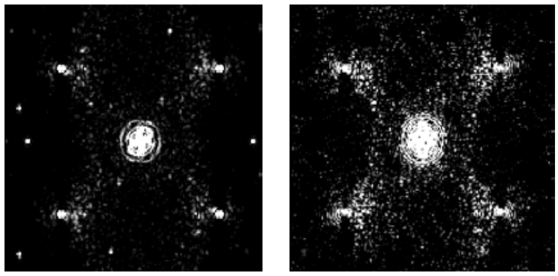
5. Conclusions

The G-P algorithm was used in order to study the possibilities of obtaining super-resolution, specially for microscopy images. It works as expected for the signal. On images, power spectrum showed restoration of frequencies, but many problems like dealing with noise and measure the obtained improvement need to be investigated to develop a new super-resolution method.



(a) (b)

Figure 2. Signal restoration: (a) clipped signal (b) restored signal



(a) (b)

Figure 3. Images power spectrum: (a) original image (b) restored image

7. Acknowledgment

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8. References

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