Models and Representation

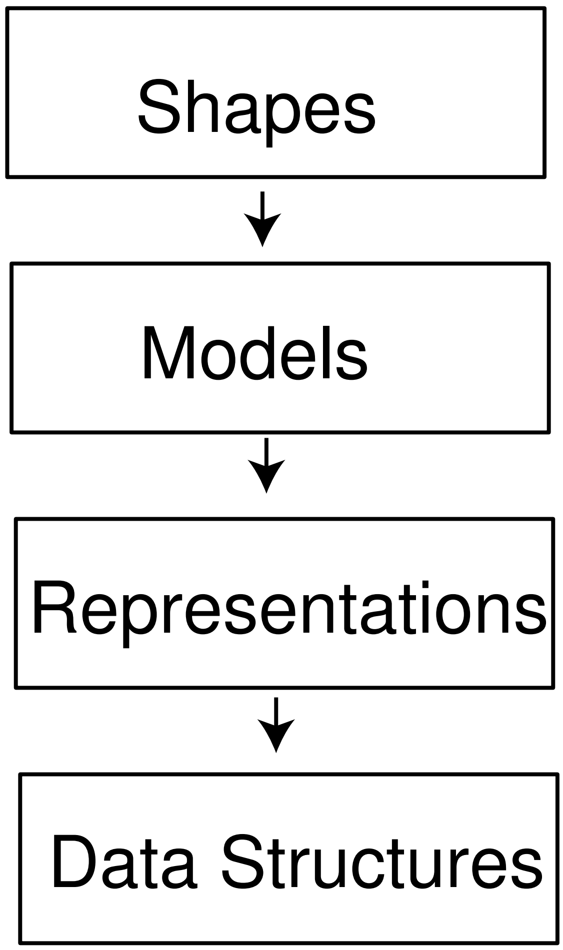
- Shape Modeling Problem
- Conceptual Framework
- Need for Solid Modeling
- Mathematical Models
- Representation Schemes

Shape Modeling

- What is a Shape?
  - Point Set in $\mathbf{R}^3$
- Types of Shapes
  - Smooth
  - Irregular
  - Fuzzy
- Describing Shapes
  - Analysis
  - Manipulation
  - Creation

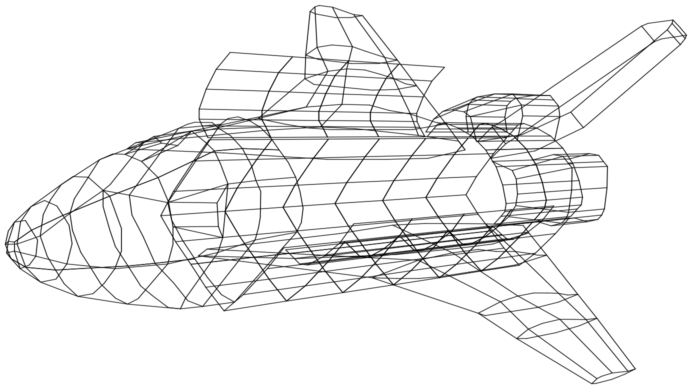
Conceptual Framework

Geometric + Procedural

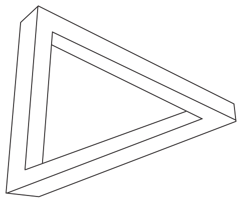


Need for Solid Modeling

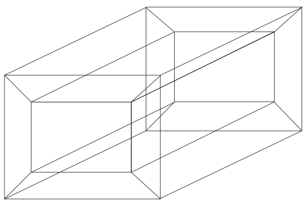
- Wire Frame Models



- Problems
  - Inconsistency



  - Ambiguity



Characterization of a Solid

- Point-Membership Classification
- Characteristic Function

$S \subset \mathbf{R}^3$

$\chi_S : \mathbf{R}^3 \to \{0, 1\}$

$$\chi(p) \begin{cases} 1 & p \in S. \\ 0 & p \notin S. \end{cases}$$

- Solid
  - Boundary (Surface)
  - Interior (Volume)

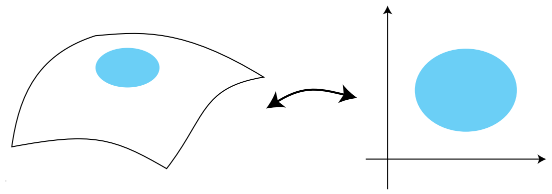
Describing a Solid

- Simple Solid
  - Classification
  - Boundary
  - Both
- Complex Solid
  - Subdivide into Simple Parts (Geometry)
  - Structure Connecting Parts (Topology)

Geometric Models

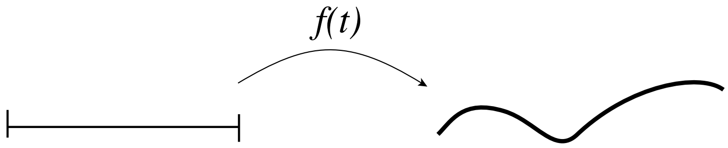
- Manifold Models

  (Homogeneous)



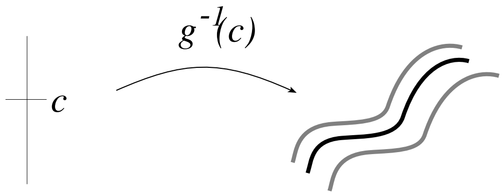
- Specification
  - Parametric

    $(x, y) = f(t)$

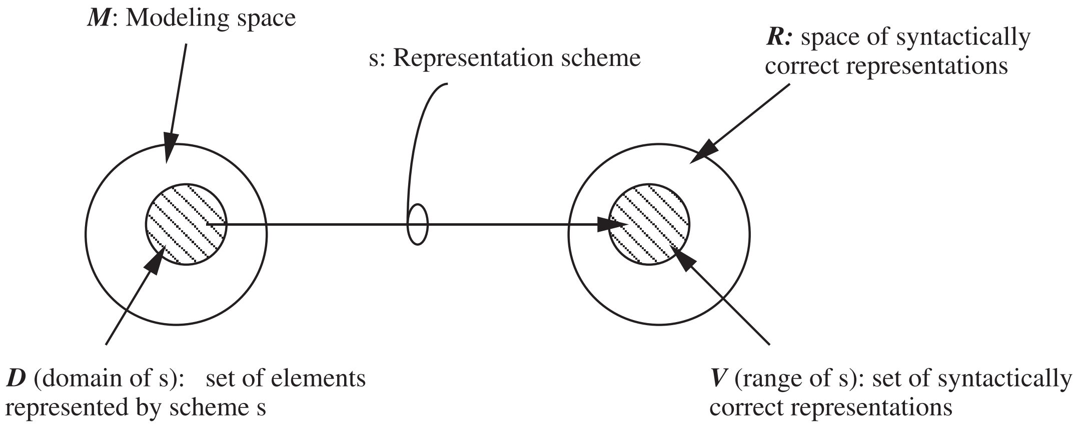


  - Implicit

    $g(x, y) = c$



Representation Schemes



$$s : M \to R$$

- Properties
  - Uniqueness
  - Ambiguity
  - Validity
- Problems
  - Conversion
  - Computability

Example: Non-uniqueness

- Ellipse

  $\frac{x-x_c^2}{a^2} + \frac{y-y_c^2}{b^2} = 1$

  M = all conics

  D = all ellipses (and circles)

  $R = \{x_c, y_c, a, b\}$

  $V = \{x_c, y_c, a, b\}$, $a, b, c > 0$

- Circle

  M = all conics

  D = all circles

  $R = \{x_c, y_c, r\}$

  $V = \{x_c, y_c, r\}$, $r > 0$

Geometric Representation Schemes

(Primitives)

- Constructive
- Decomposition

Pure Primitive Representation

- Shape Library
- Generation Operator
- Transformations

Example: Sphere

- Parametric Model

  $(x, y) = (\cos t, \sin t)$

- Implicit Model

  $x^2 + y^2 = 1$

- Representation

  id, center, radius

Constructive Representation

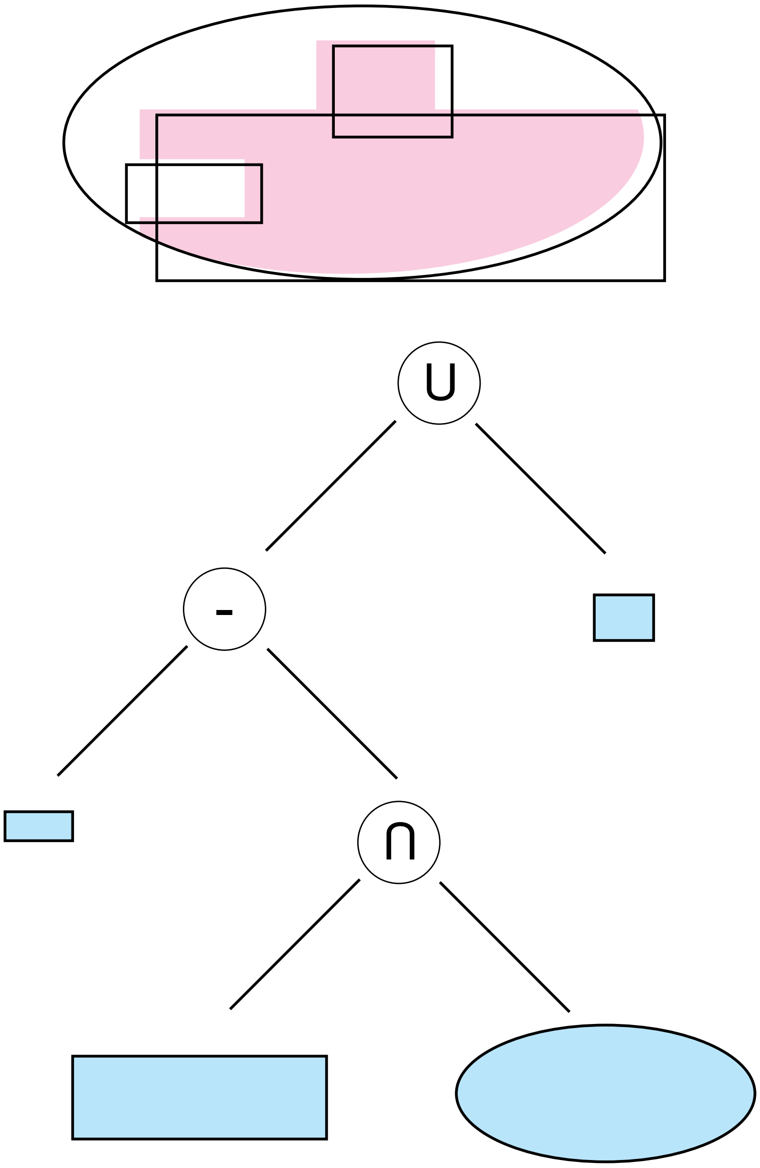
- Half Space Elements
- R-Set Operations

  $\cup$ – union

  $\cap$ – intersection

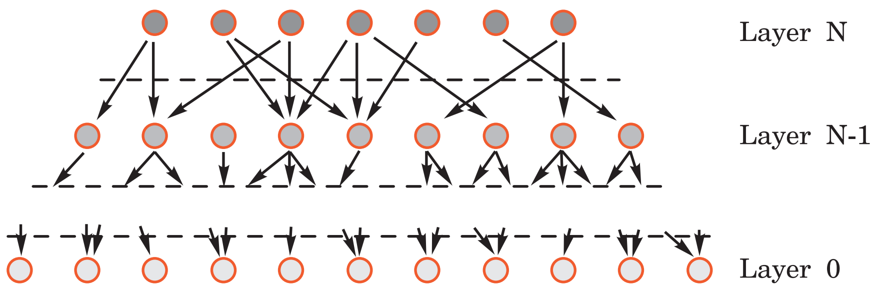
  $\setminus$ – difference
- CSG Tree

Example: CSG Quadrics



Decomposition Representation

- Stratification
  - Vertices
  - Edges
  - Faces
  - Shells
- Topological Graph
  - Layers
  - Links

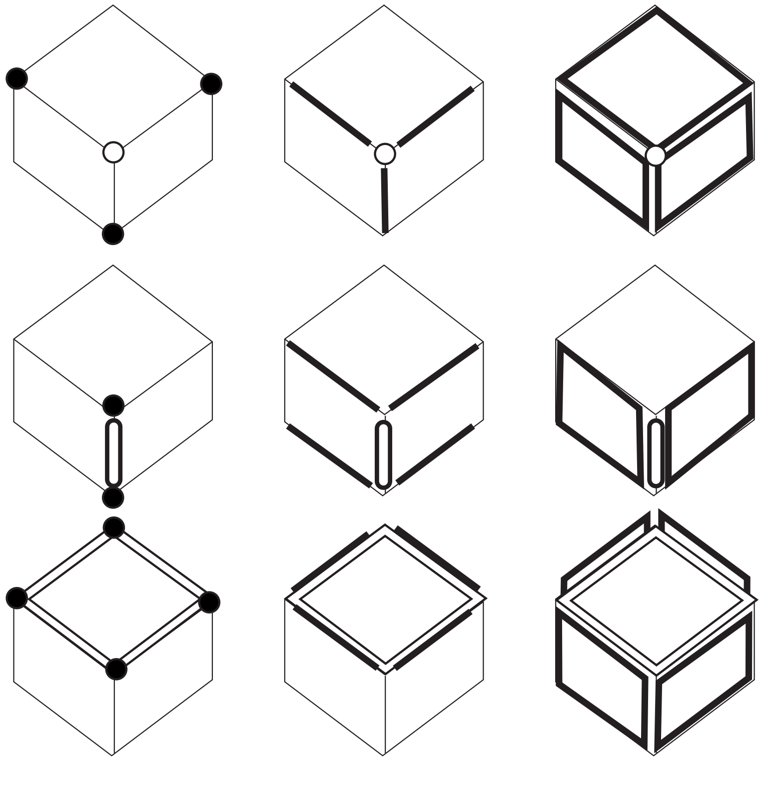


2 Manifold Topology

- Elements

  V, E, F

- Adjacency Relations



Vertex-Polygon Structure

- Vertices

  $V_1 = (x_1, y_1, z_1)$

  $V_2 = (x_2, y_2, z_2)$

  $\ldots$

  $\ldots$

  $V_n = (x_n, y_n, z_n)$

- Loops

  $L_1 = \{V_{1a}, V_{1b}, V_{1c}\}$

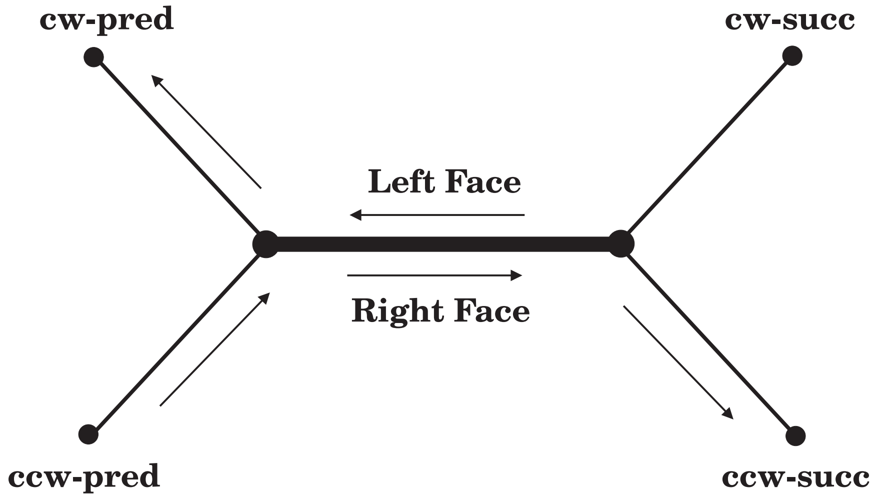
  $L_1 = \{V_{2a}, V_{2b}, V_{2c}\}$

  $\ldots$

  $\ldots$

  $L_n = \{V_{Na}, V_{Nb}, V_{Nc}\}$

Winged Edge Structure



Example: Polyhedral Model

- Model
  - Cell Decomposition
- Geometry
  - Piecewise Linear
- Representation
  - B-Rep
- Data Structure
  - Vertex-Polygon

Auxiliary Representations

- Approximate Descritpions
  - Uniform
  - Adaptive
- Spatial Decomposition
  - Neighborhood Geometry
  - Spatial Hash Table
- Types
  - Voxel Arrays
  - N-Trees

Example: Octree

- insert fig