
Mesh Refinement Methods

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Outline

- Refinement Schemes
 - Subdivision Templates
 - Insertion Patterns
 - Computational Framework
 - Basic Algorithm
 - Sampling Criteria and Mesh Consistency
 - Base Mesh Generation
 - Parametric and Implicit Surfaces
 - Sampled Data
 - Uniform Refinement
 - Edge and Vertex Based
 - Non-Uniform Refinement
 - Restricted and Edge Coherence
 - Hybrid Refinement
-

Intuition

- General Setup
 - Start with a Coarse Representation
 - Apply a Refinement Scheme
 - Types of Input Description
 - Analytic Function
(Parametric or Implicit)
 - Sample Points
(Sparse Data)
-

Refinement Schemes

- Edge Splitting
 - Split Edges in Two Sub-Edges
 - Retriangulate using a Template

good for analytic surfaces

- Vertex Insertion
 - Insert New Vertices inside a Triangle
 - Retriangulate (Delaunay)

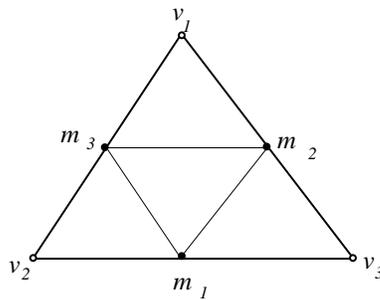
good for sampled data

- Hybrid Schemes
-

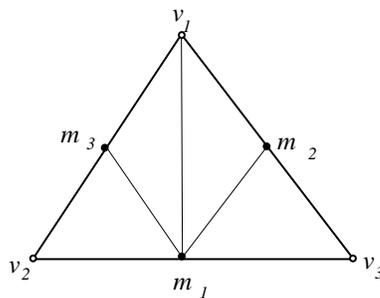
Uniform Decomposition

Split all 3 edges of a Triangle

- Isotropic Template



- Anisotropic Template

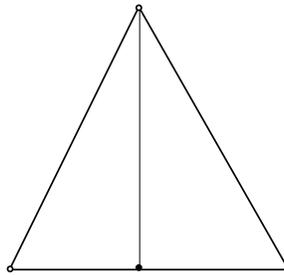


* *Three different orientations*

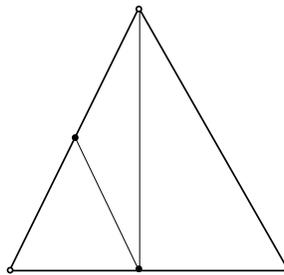
Adaptive Decomposition

Split only some edges of a Triangle

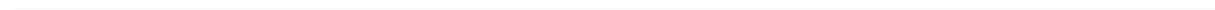
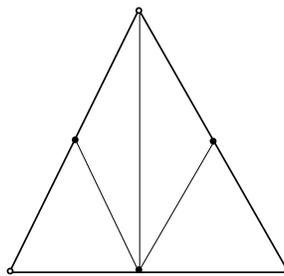
- Subdivision Templates
 - One Edge Split



- Two Edges Split

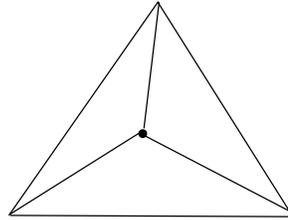


- Three Edges Split

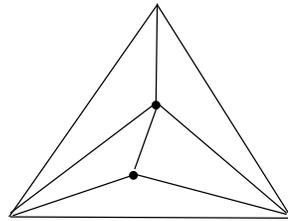


General Decompositions

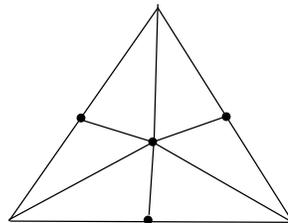
- Ternary Subdivision
(one vertex insertion)



- Restricted Delaunay Subdivision
(more than one vertex inserted)



- Edge-Vertex Subdivision
(Hybrid)



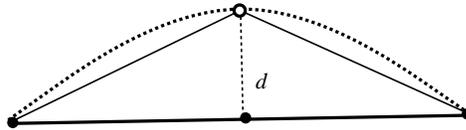
Computational Framework

- Basic Operations
 - Sampling
 - Structuring
- Scope of Execution
 - Domain Wide
 - Element Wide
- Classes of Algorithms
 - Mesh Subdivision
 - Insertion-Retriangulation

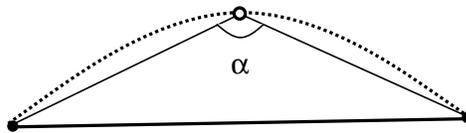


Sampling Criteria

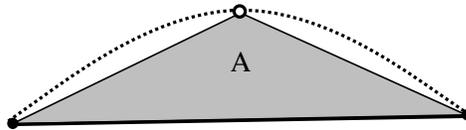
- Distance



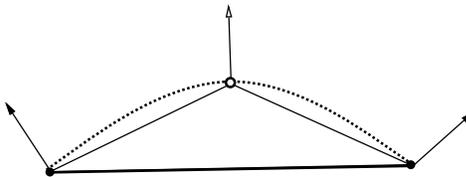
- Angle



- Area

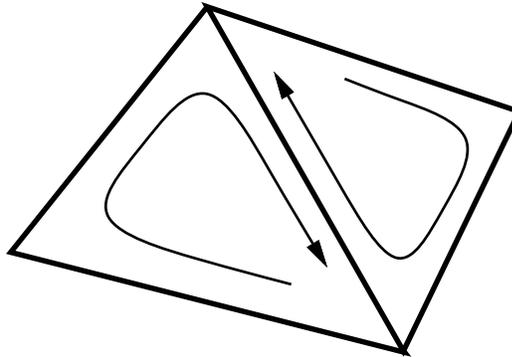


- Normal

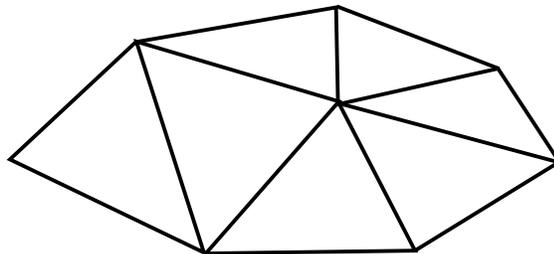


Maintaining Mesh Consistency

- Loop Orientation



- Simplicial Complex

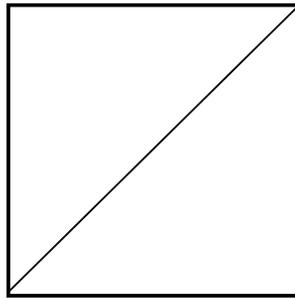


Mesh Subdivision Method

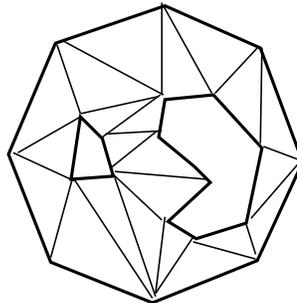
- Basic Algorithm
 1. **Initialization:**
 - Start with a Base Mesh
 2. **Refinement:**
 - Recursively subdivide cells according to sampling criteria
 - Subdivision
 - Uniform
 - Adaptive
 - Surface Type
 - Parametric
 - Implicit
 - Output Mesh
 - Single
 - Hierarchical
-

Base Mesh Generation of Parametric Surfaces

- Rectangular Domain
 - Regular Grid / CFK Triangulation



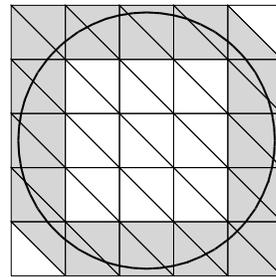
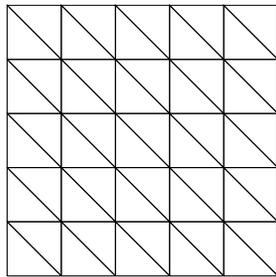
- Arbitrary (Trimmed Domain)
 - Constrained Delaunay Triangulation



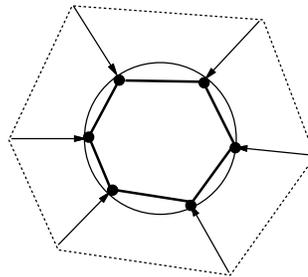
- * OBS: *Problems*
 - Embedding vs. Minimal Resolution
 - Topology
-

Base Mesh Generation of Implicit Surfaces

- Uniform Subdivision of Space
 - 3D CFK decomposition + Polygonization



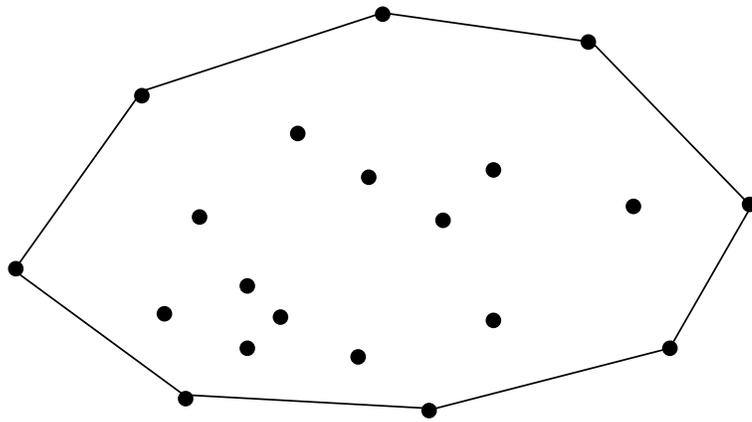
- Projecting a Polyhedron
 - Physics Based Method



- * OBS: *Problems*
 - Tubular Neighborhood
-

Base Mesh Generation of Sampled Data

- Simplest Mesh
 - Convex Hull
 - Internal Triangulation

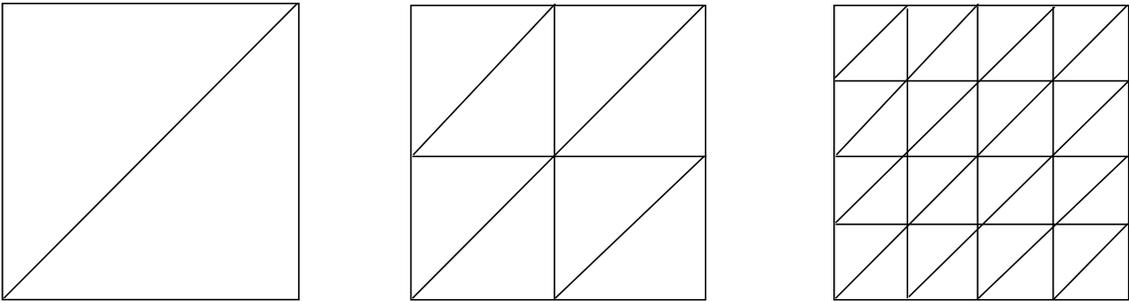


* Delaunay Triangulation

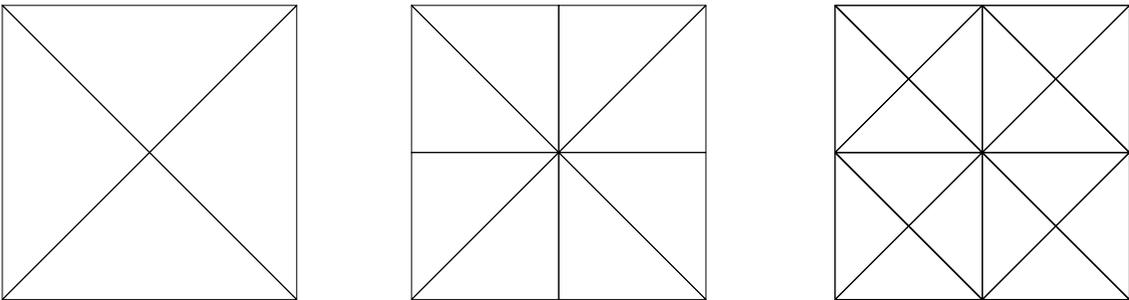
Uniform Refinement Methods

Subdivide *all* Cells Recursively

- Isotropic



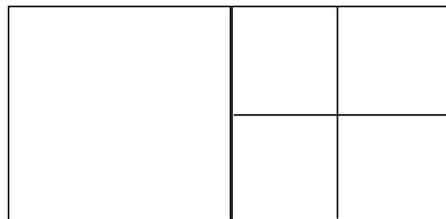
- Anisotropic



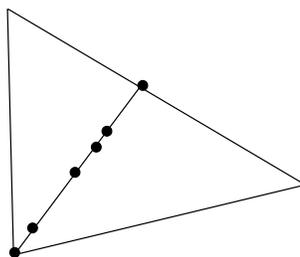
Non-Uniform Refinement Methods

Subdivide *only* the Cells that don't satisfy adaptation criteria

- Restricted
 - hierarchy with one-level difference



- Edge Coherence
 - Same sampling criteria
 - Completely sample edges



Case Study: HAP

1. Initialization

1.1 Generate Initial Mesh

1.2 Sample Edges

2. Refinement

For each element

 If All Edges not Simple

 Decompose Element

 Output Sub-elements

 Else

 Output Element

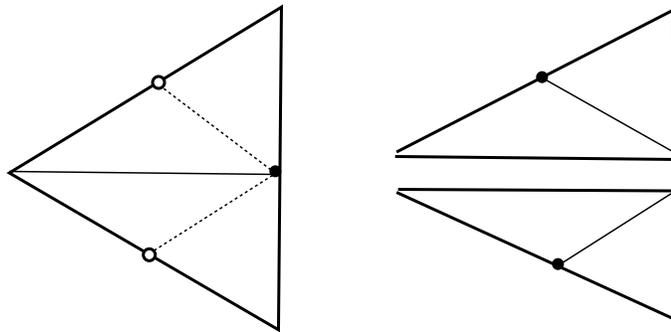
OBS: Adaptation

– Adapted Subdivision

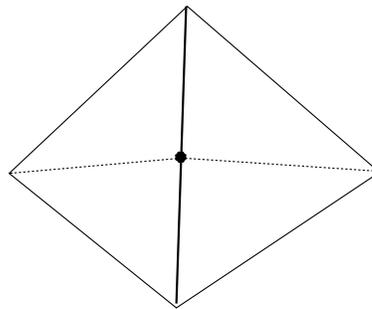
– Uniform Subdivision

HAP Subdivision Scheme

- Two-Step Decomposition
 1. Split along *Best* Direction
 - Error
 - Aspect Ratio
 2. For each side
 - If opposite edge is Complex
 - Split



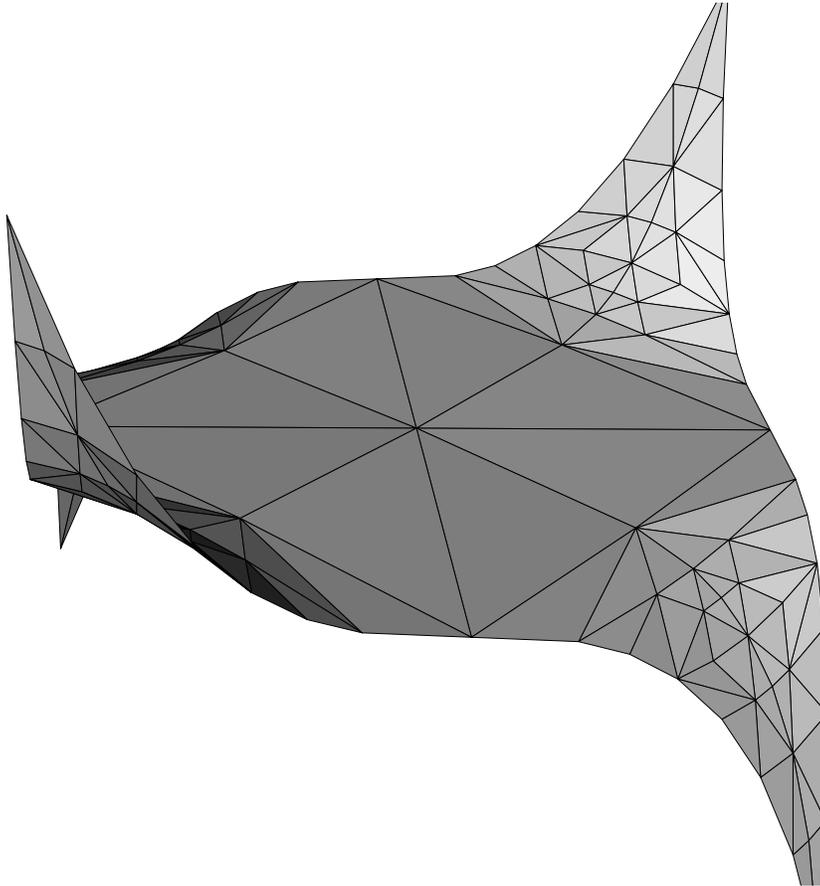
- Fragments
 - Split-Edge



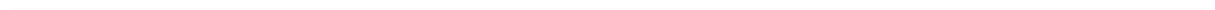
Examples

- Adapted Refinement
 - Hierarchical Decomposition
 - Comparison: Parametric / Implicit
 - Complicated Topology / Domain
 - Rendering
-

Parametric Cubic Surface

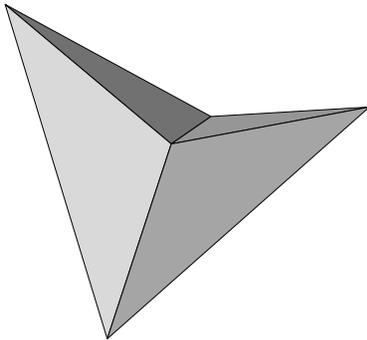


$$\begin{aligned}x &= u, & y &= v, & z &= (uv)^3 \\ u &\in [0, 1], & v &\in [0, 1].\end{aligned}$$

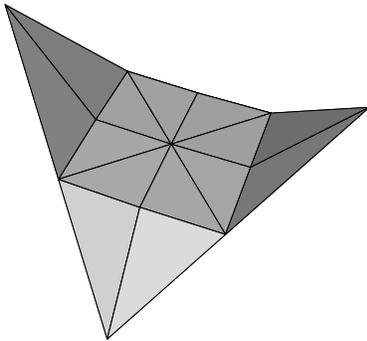


Refinement Hierarchy

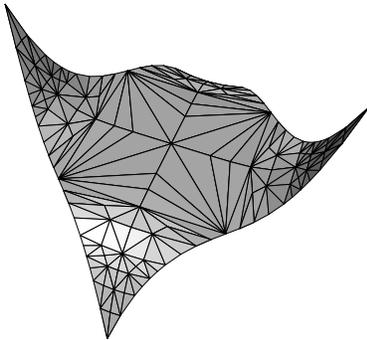
Level 1



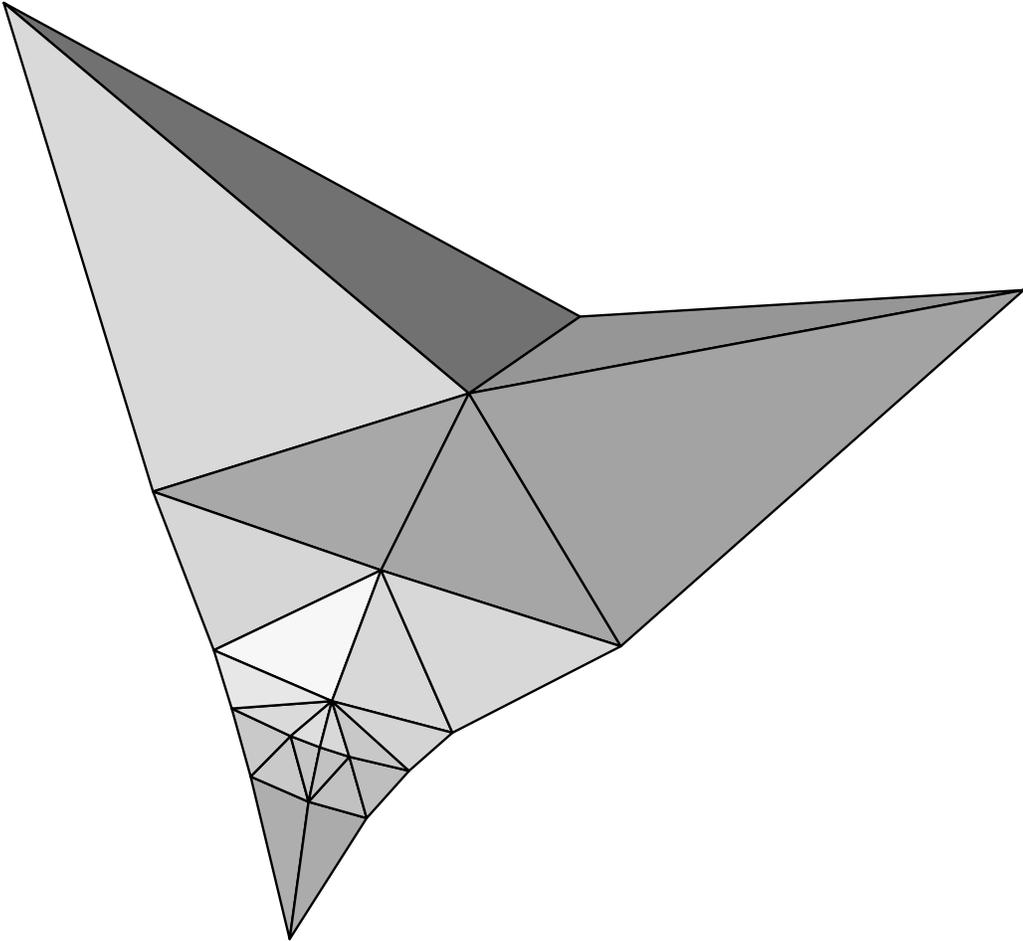
Level 3



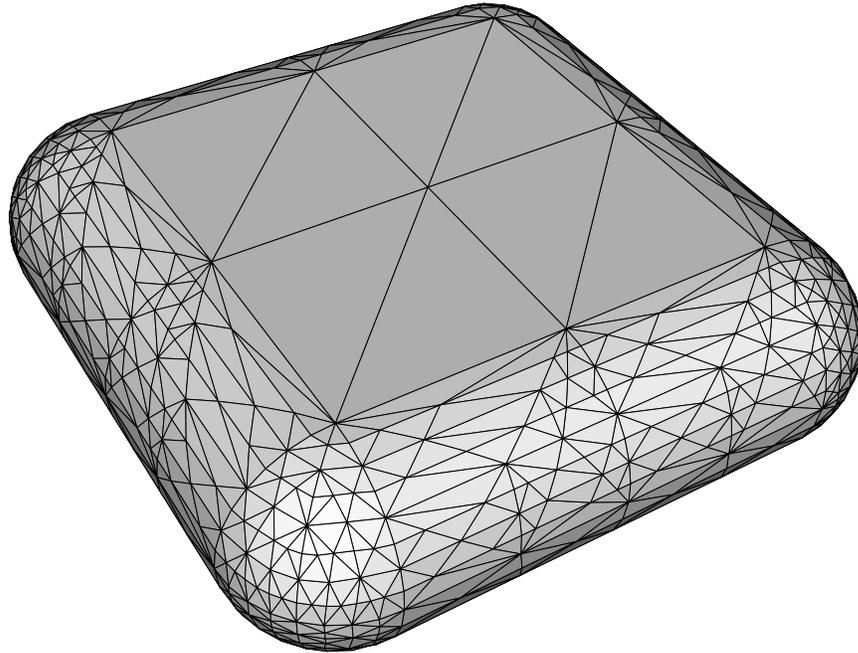
Level 5



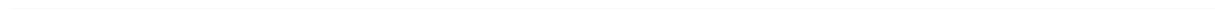
Variable Resolution Mesh



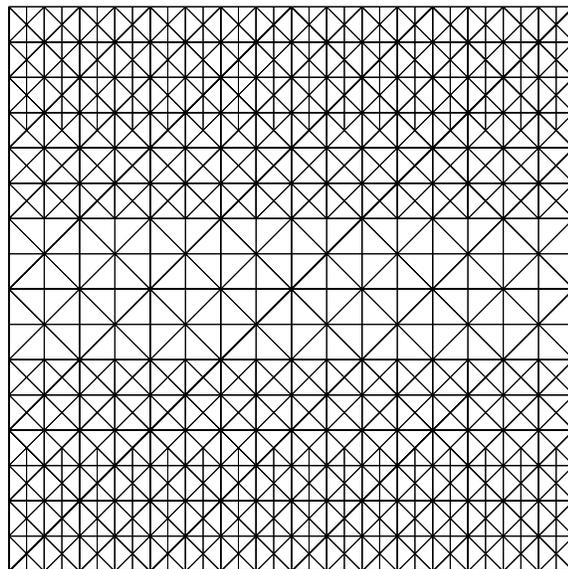
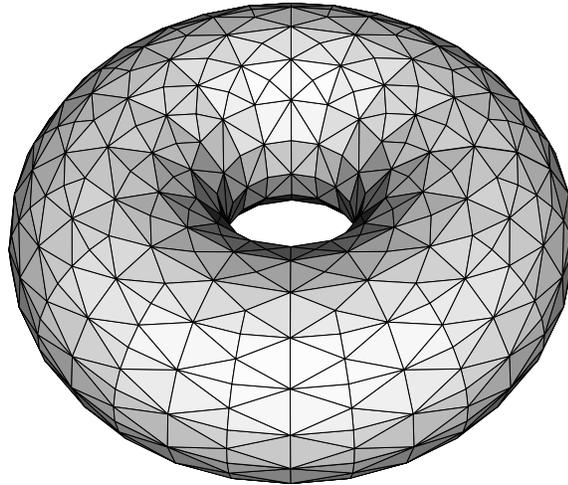
Implicit Offset Surface



$$f(x, y, z) = \text{dist}(x, y, z, S) - r$$
$$S = [0, 1] \times [0, 1] \times \{0\} \subset \mathbf{R}^3, \quad r = 0.25.$$

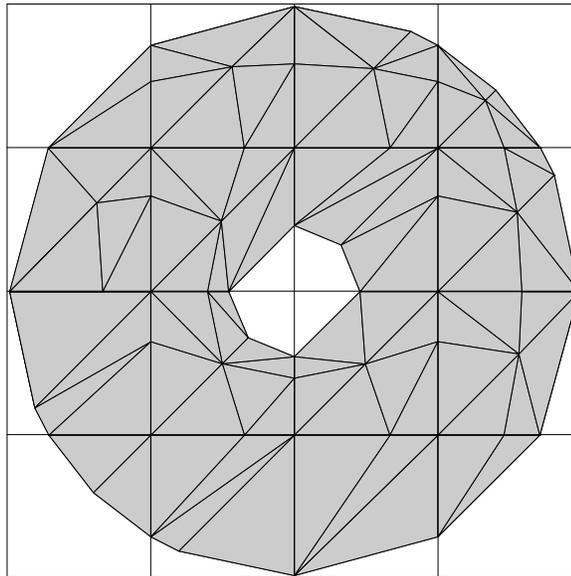
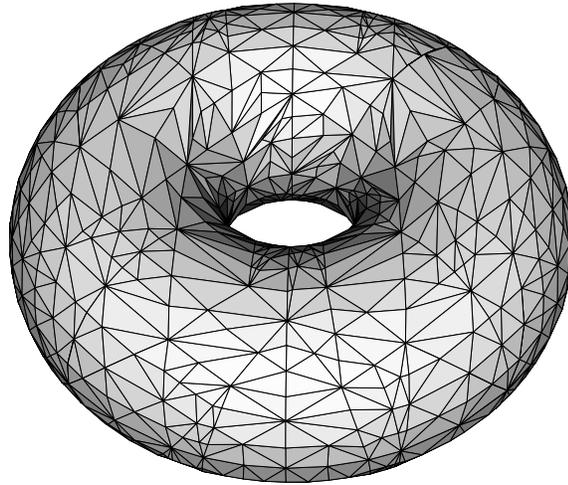


Parametric Torus



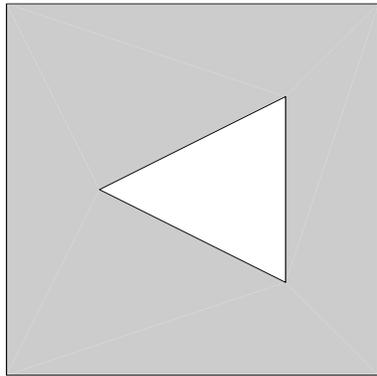
$$\begin{aligned}x &= \cos u(r + \cos v) & y &= \sin u(r + \cos v) & z &= \sin v \\u &\in [0, 2\pi], & v &\in [0, 2\pi], & r &= 1.6.\end{aligned}$$

Implicit Torus

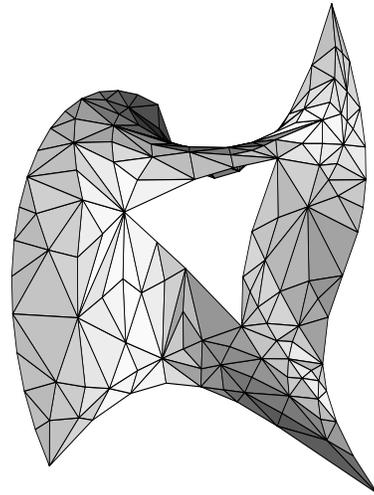


$$f(x, y, z) = (x^2 + y^2 + z^2 - r^2 - 1)^2 - 4r^2(1 - z^2)$$
$$x, y \in [-3, 3], \quad z \in [-1, 1], \quad r = 1.6$$

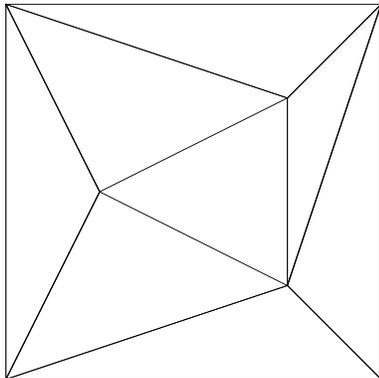
Complicated Domain: Trimmed Bezier Patch



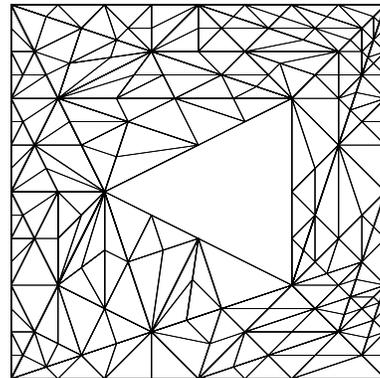
uv domain



3D surface



base mesh



final mesh

Levels of Detail and Rendering

