A Manifold-Based Construction for Surface Fitting

Lecture 5 - February 5, 2009 - 1-2 PM

Outline

Outline

- The Surface Fitting Problem
- Building a Set of Gluing Data
- Conclusions
- Suggested readings

The Surface Fitting Problem

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Given a mesh S_T in $\mathbb{R}^3$, a positive integer k, and a positive real number ϵ, our goal here is to fit a C^k surface, S, in $\mathbb{R}^3$ to S_T.

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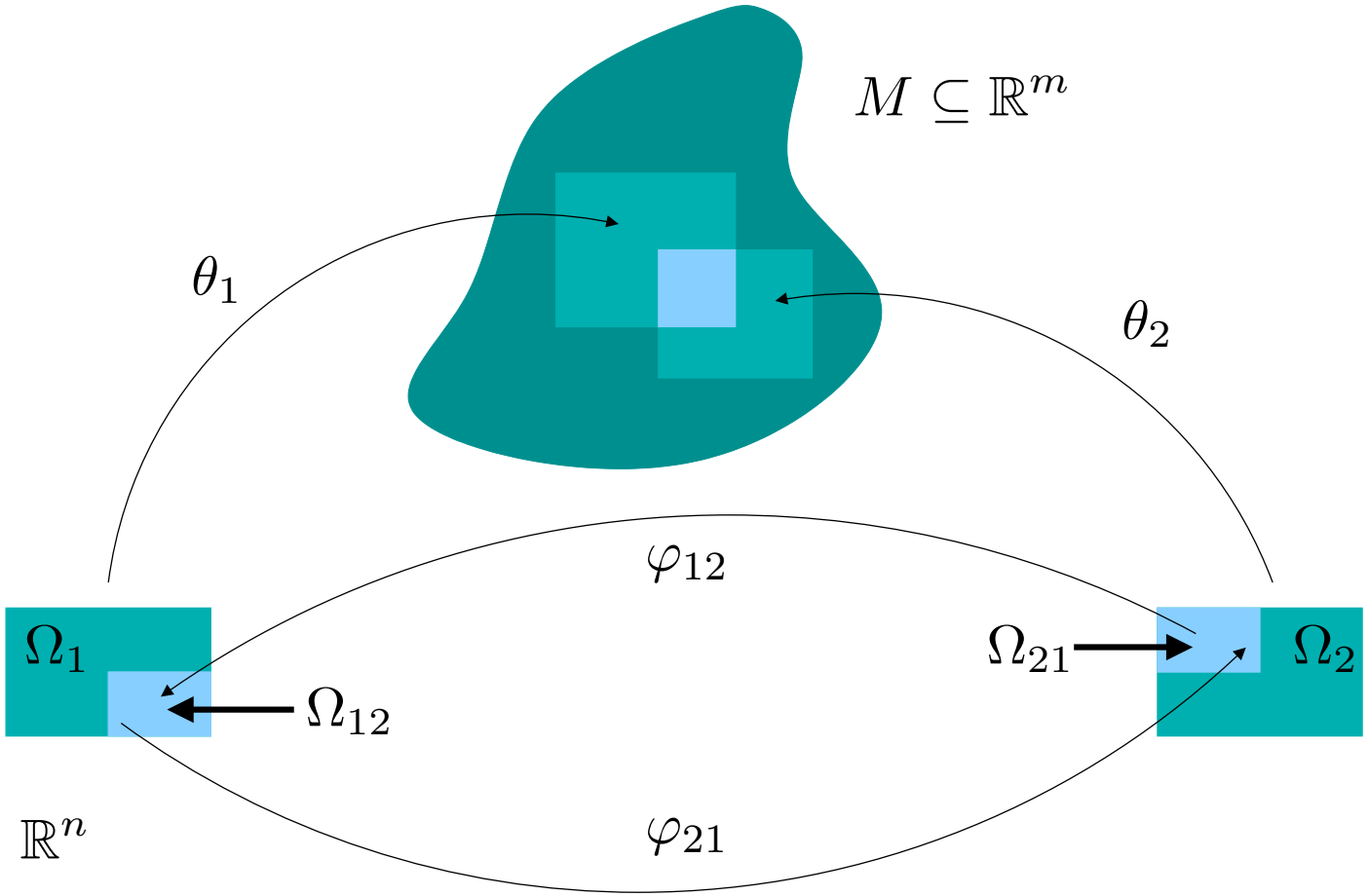
The Manifold-Based Approach:

We solve the fitting problem by defining a C^k parametric pseudo-surface, $\mathcal{M}$, such that S is the image, M, of $\mathcal{M}$ in $\mathbb{R}^3$.

The Surface Fitting Problem

The Surface Fitting Problem

Big Picture



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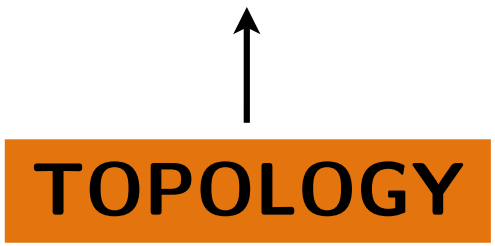

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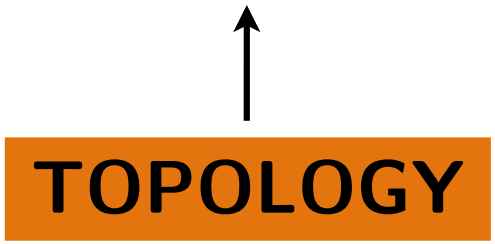

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The Surface Fitting Problem

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TOPOLOGY

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GEOMETRY

Building a Set of Gluing Data

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$$\mathcal{G} = \left((\Omega)_{i\in I}, (\Omega_{ij})_{(i,j)\in I\times I}, (\varphi_{ij})_{(i,j)\in K}\right)$$

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- Vecchia, Jüttler, and Kim, CAGD (to appear).

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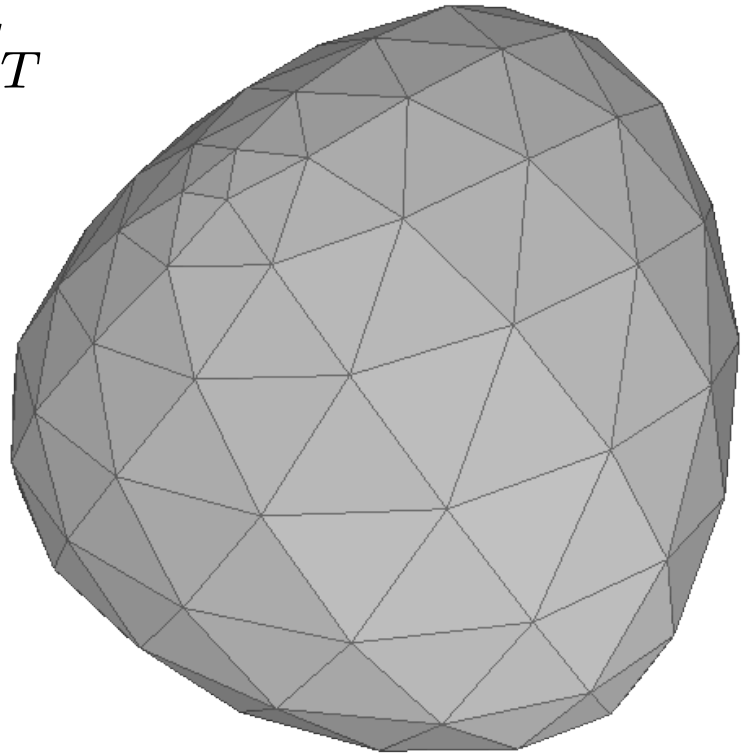
S_T



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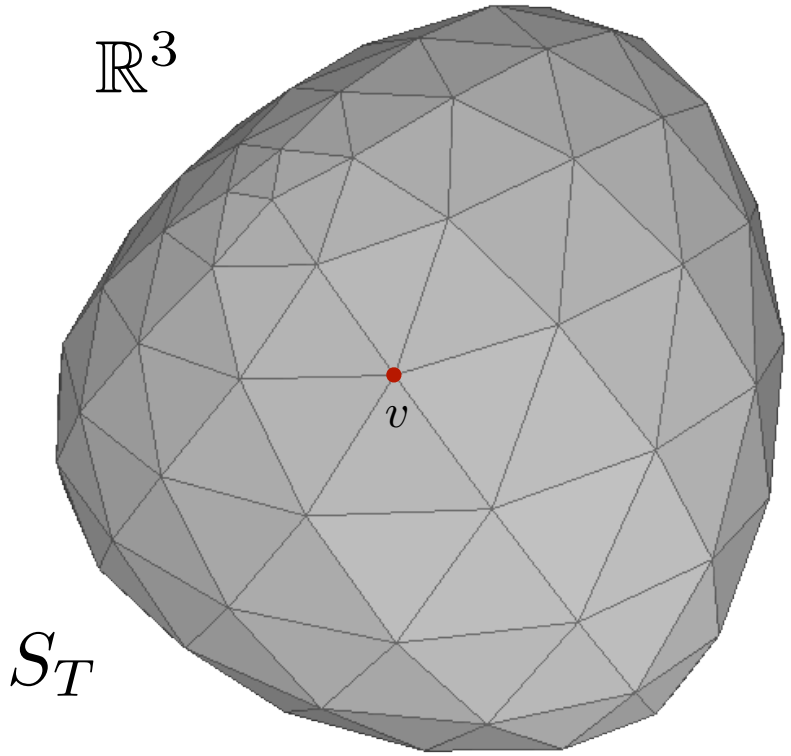

Building a Set of Gluing Data

Building a Set of Gluing Data

Let

$$l : I \to \{1, \ldots, |I|\}$$

be a map that assigns a unique *label* from $\{1, \ldots, |I|\}$ to each vertex in I.

Building a Set of Gluing Data

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For every v in I, the p-**domain**, Ω_v, is the point set

$$\Omega_v = \{(x, y) \in \mathbb{R}^2 \mid (x - 2 \cdot l(v))^2 + y^2 < [\cos(\pi/m_v)]^2\},$$

where m_v is the degree of v.

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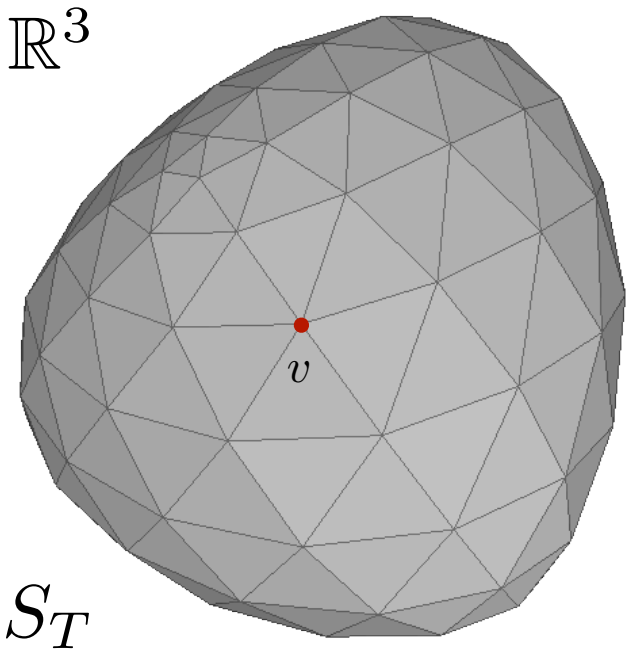

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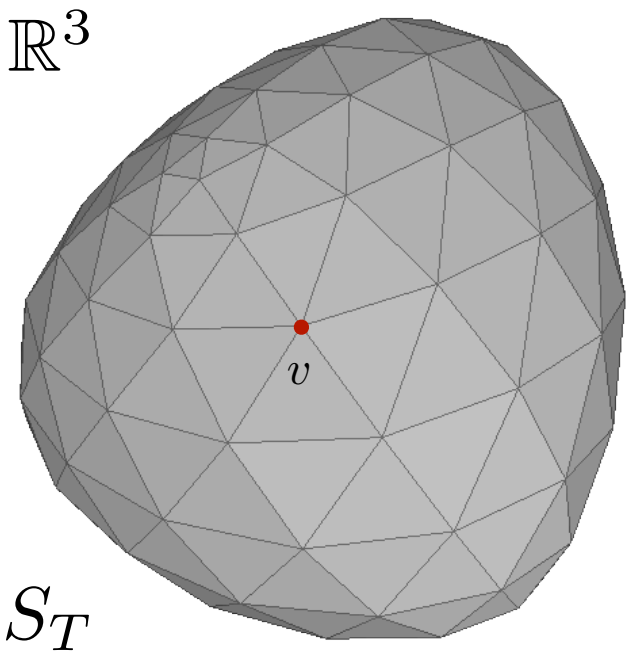

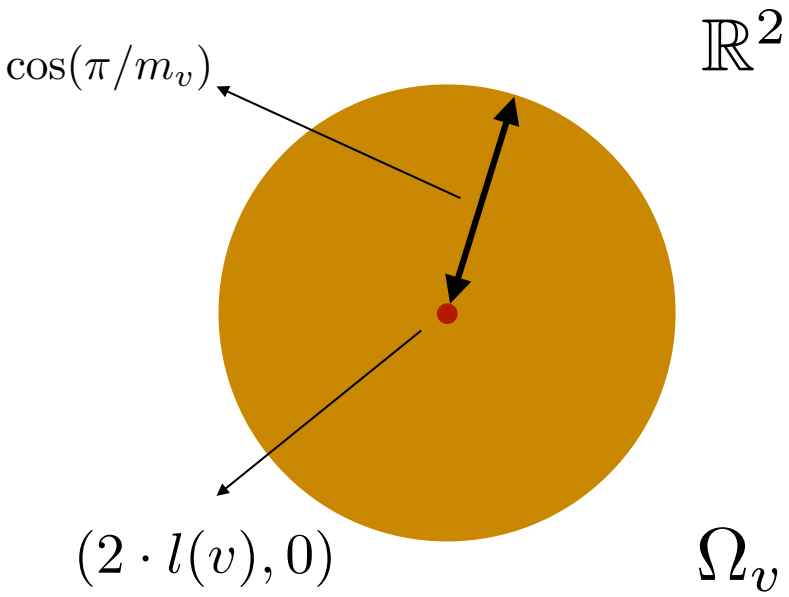

Building a Set of Gluing Data

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Remark:

It is important to mention that for every $v \in I$, the set Ω_v is open in $\mathbb{R}^2$. Furthermore, for every $u, w \in I$, with $u \neq w$, we have

$$\Omega_u \cap \Omega_w = \emptyset \,.$$

So, the $(\Omega_v)_{v \in I}$ is a family of p-domains (see definition of sets of gluing data).

Building a Set of Gluing Data

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Gluing domains

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Gluing domains

To define the family of gluing domains, $(\Omega_{ij})_{(i,j)\in I\times I}$, we need define the notions of a P-polygon and its associated triangulation, two linear functions, and a more general, transcendental map.

Building a Set of Gluing Data

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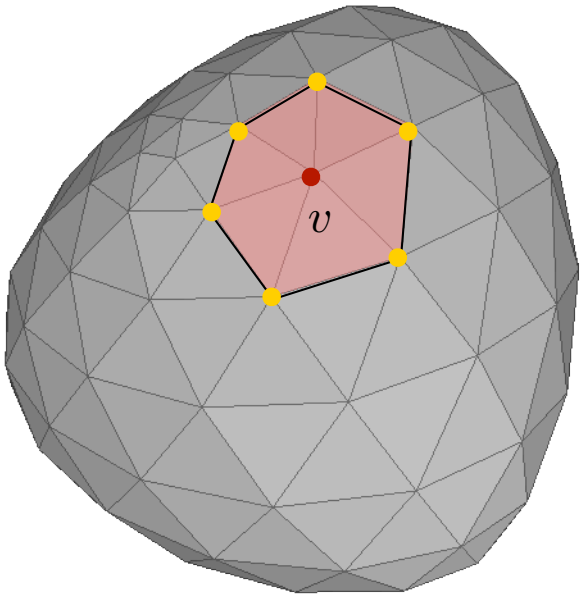
For each vertex v of S_T, we define a *P-polygon*, P_v:

$\mathbb{R}^3$



S_T

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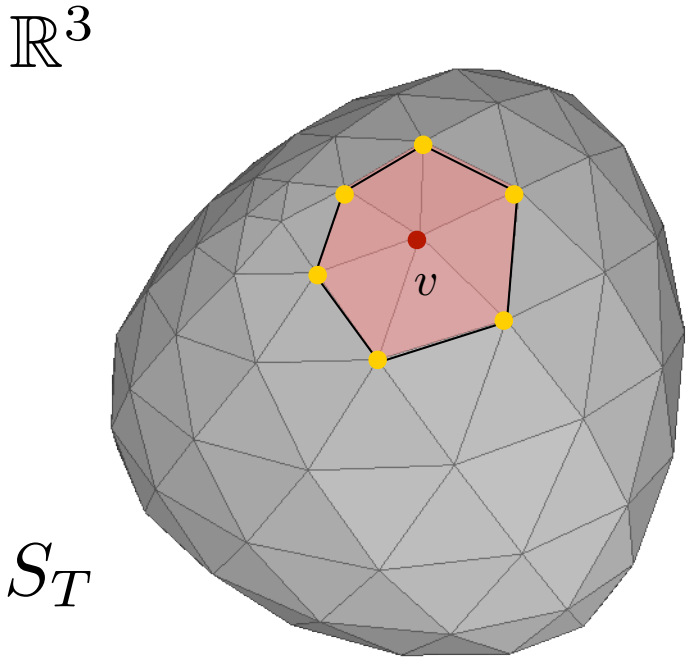

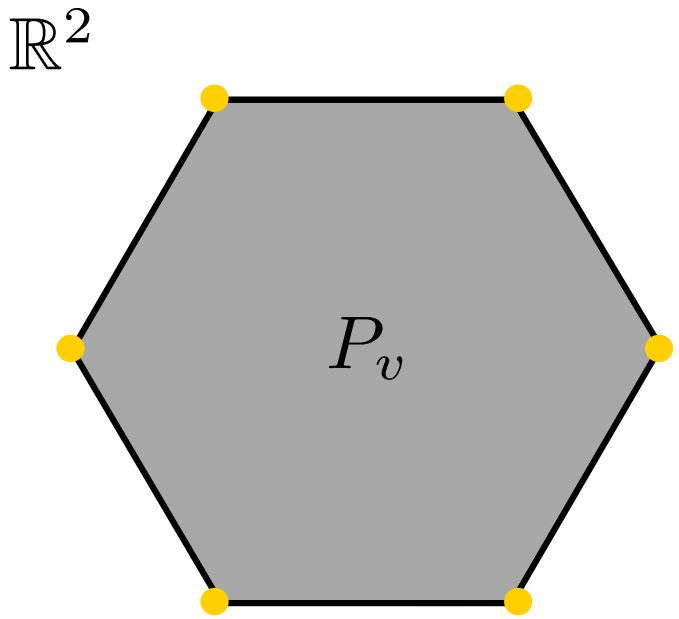

Building a Set of Gluing Data

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For each vertex v of S_T, we define the **P-polygon, P_v, associated with** v as the regular polygon in $\mathbb{R}^2$ given by the vertices

$$v_i' = \left(2 \cdot l(v) + \cos\left(\frac{2\pi \cdot i}{m_v}\right), \sin\left(\frac{2\pi \cdot i}{m_v}\right)\right),$$

for each $i \in \{0, \ldots, m_v - 1\}$, where m_v is the degree of v.

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For each vertex v of S_T, we define the **P-polygon, P_v, associated with** v as the regular polygon in $\mathbb{R}^2$ given by the vertices

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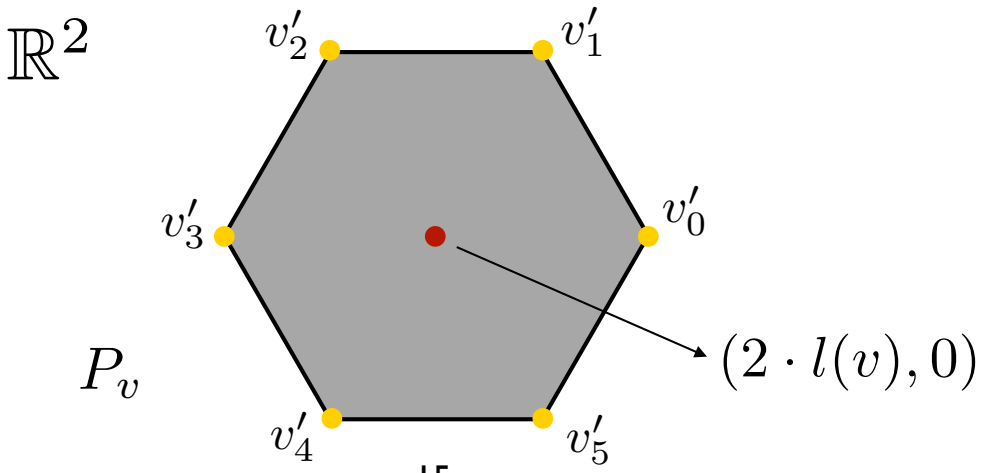

Building a Set of Gluing Data

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Let C_v be the circle inscribed in P_v. Then, C_v is the point set

$$C_v = \{(x, y) \in \mathbb{R}^2 \mid (x - 2 \cdot l(u))^2 + y^2 \leq [\cos(\pi/m_v)]^2\},$$

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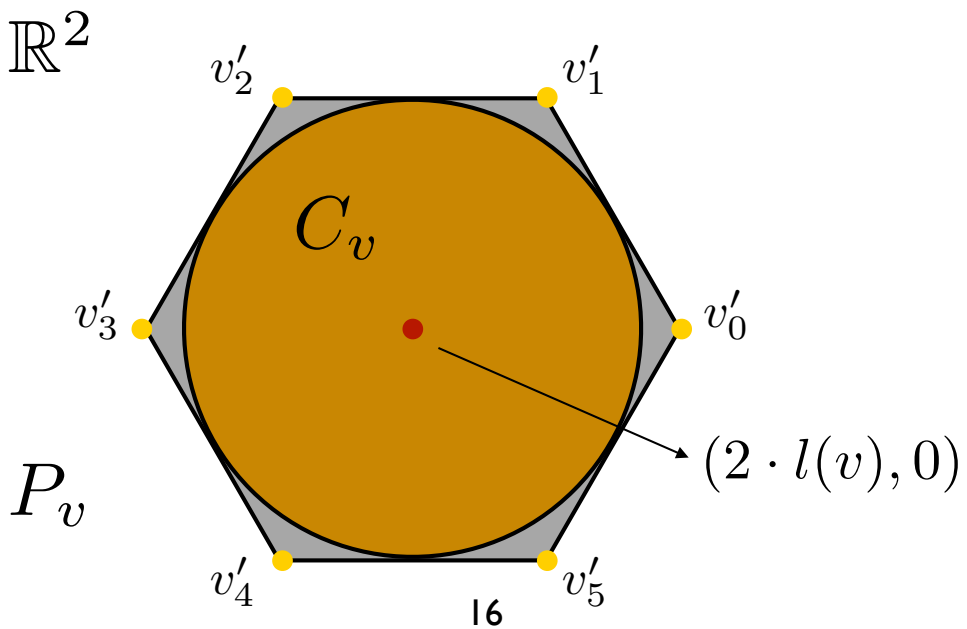

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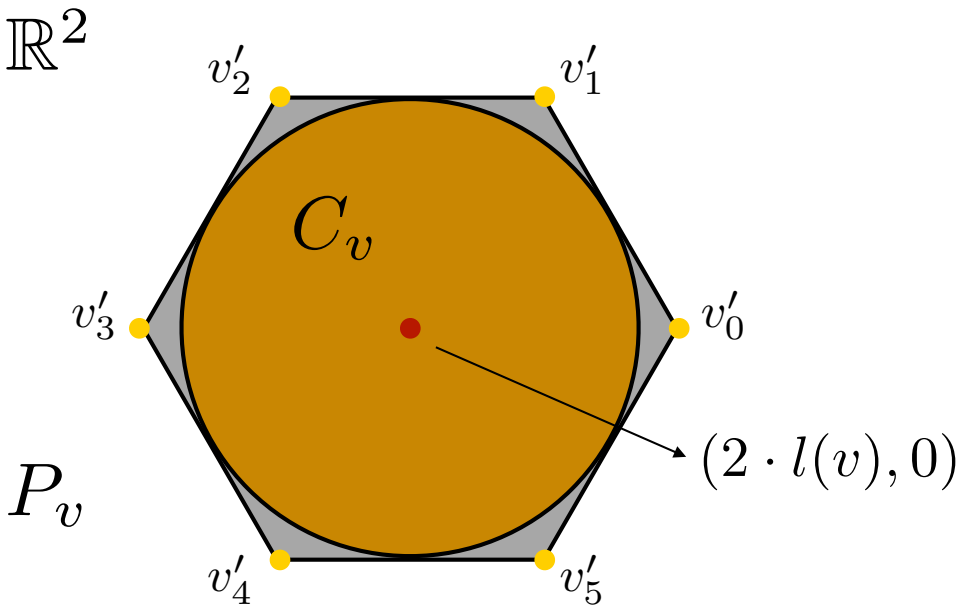

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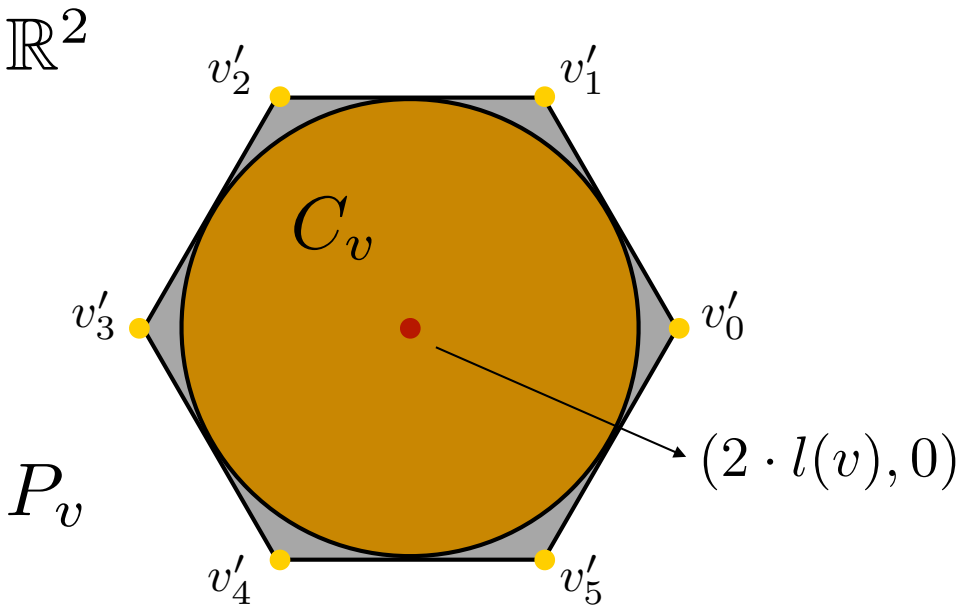

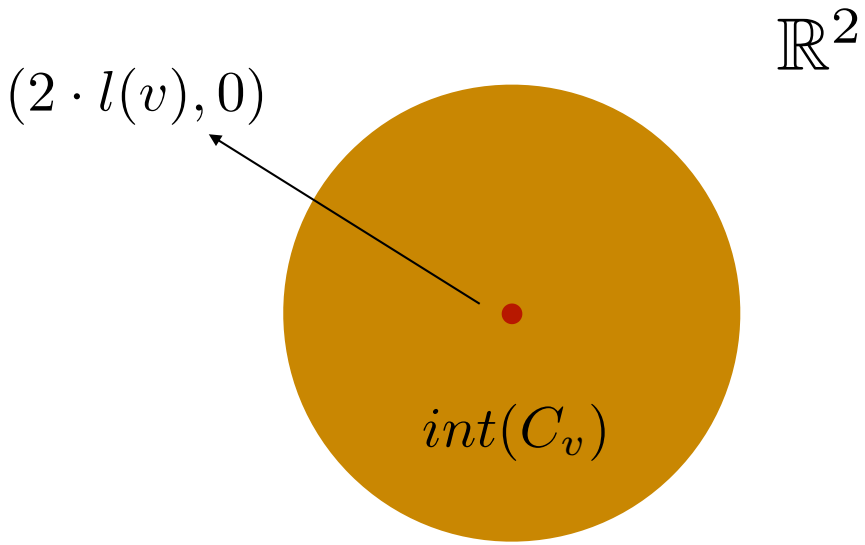

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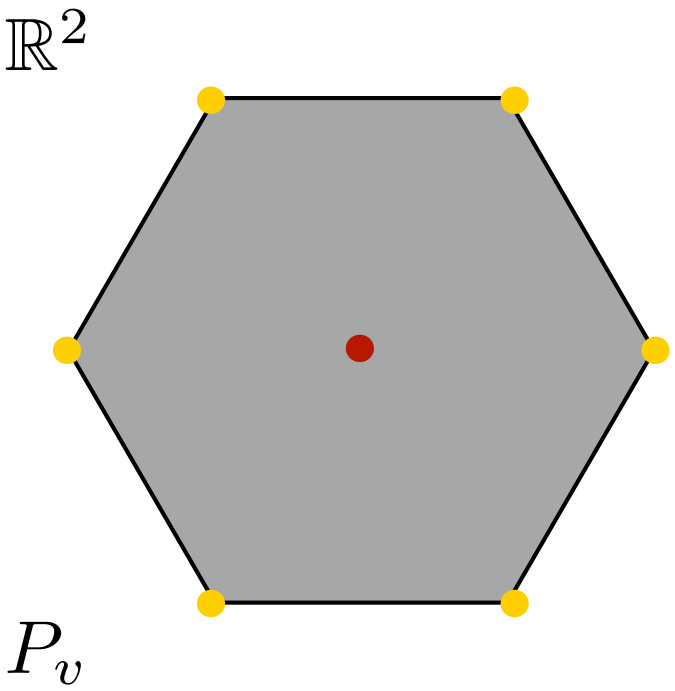

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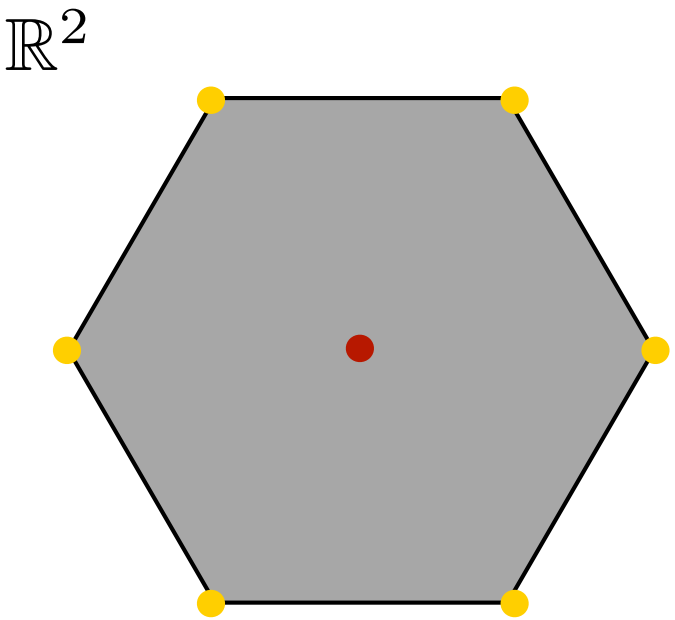

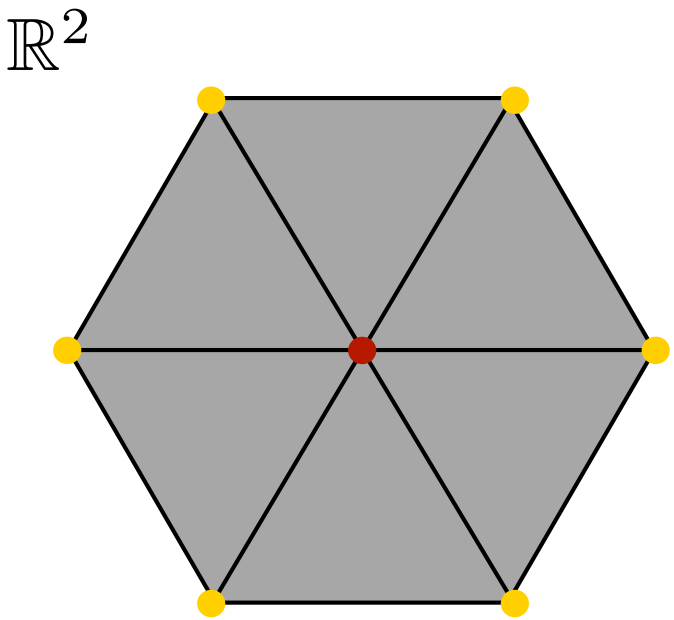

Building a Set of Gluing Data

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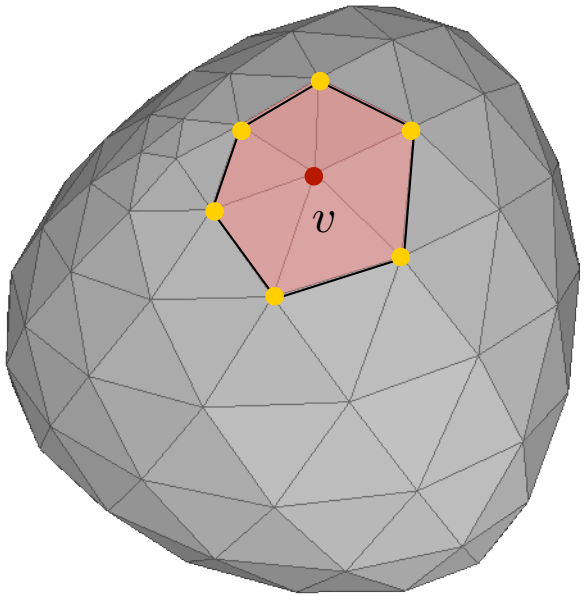
For every vertex, v, of S_T, consider its **star**, $st(v, S_T)$:

$\mathbb{R}^3$



S_T

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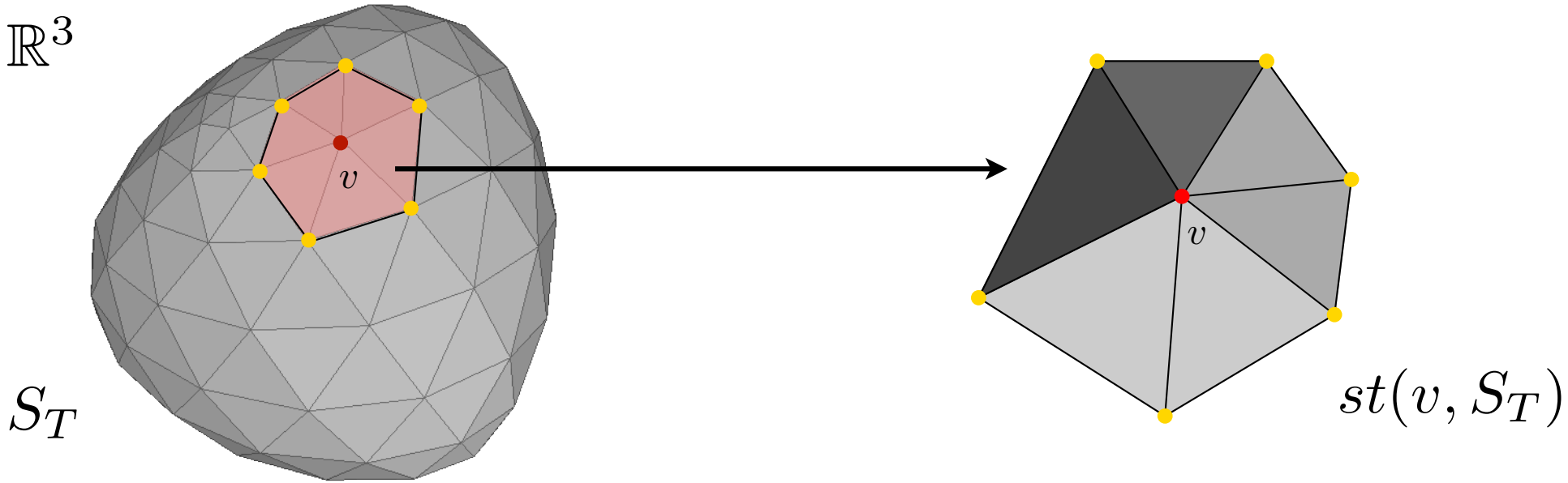

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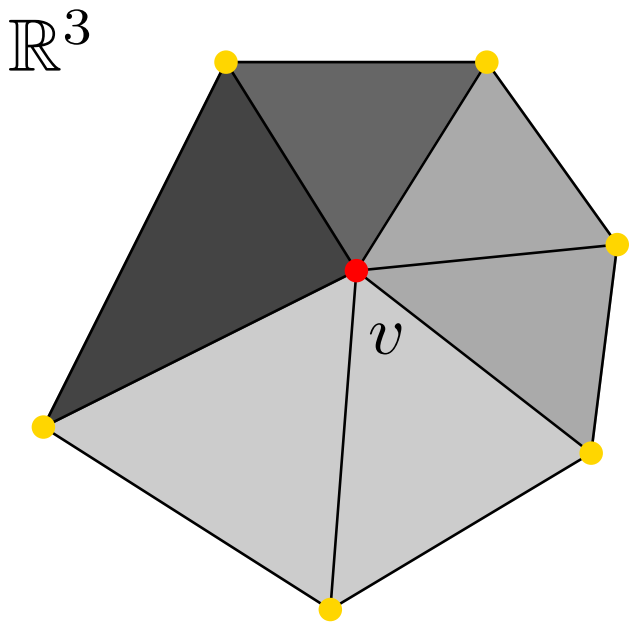
Note that T_v can be viewed as a parametrization of $st(v, S_T)$ in $\mathbb{R}^2$:



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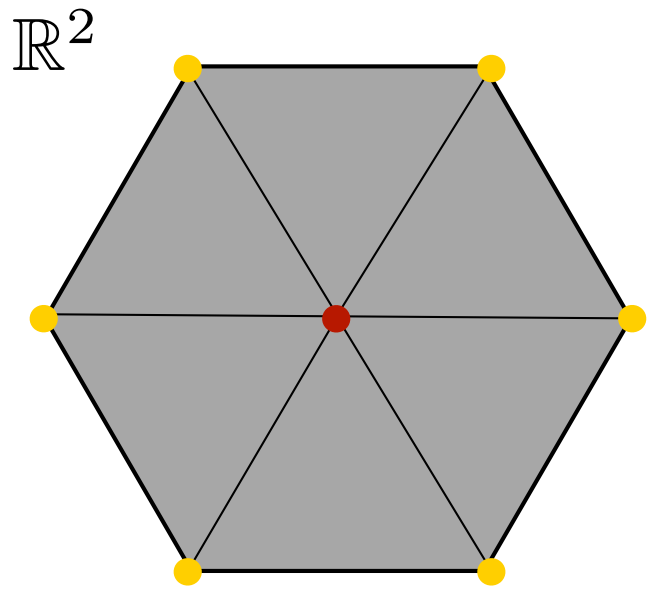


T_v

Building a Set of Gluing Data

Building a Set of Gluing Data

Indeed, we can define a bijection,

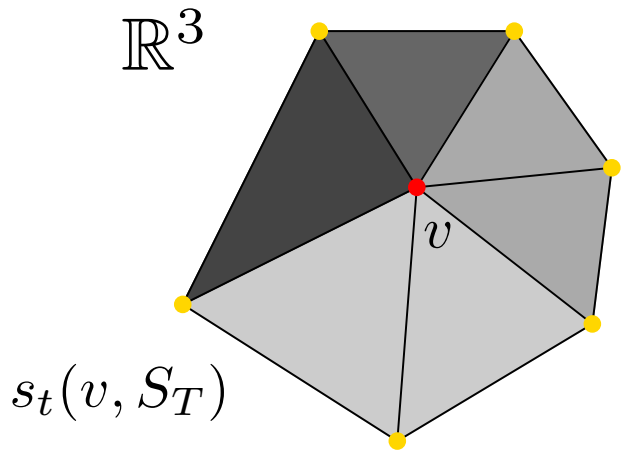
$$s_v : st(v, S_T) \to T_v \,,$$

which maps vertices (resp. edges and triangles) of $st(v, S_T)$ to vertices (resp. edges and triangles) of T_v, as described in the next slide:

Building a Set of Gluing Data

Building a Set of Gluing Data

To define s_v, it suffices to make a one-to-one correspondence between the vertex sets of $st(v, S_T)$ and T_v such that (1) $s_v(v) = v'$ and



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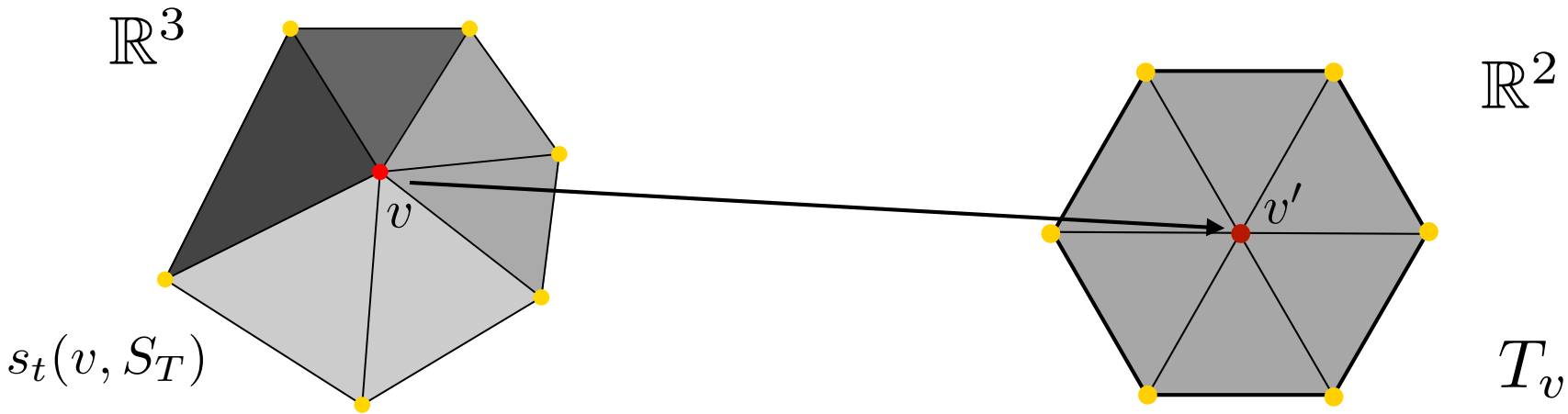

Building a Set of Gluing Data

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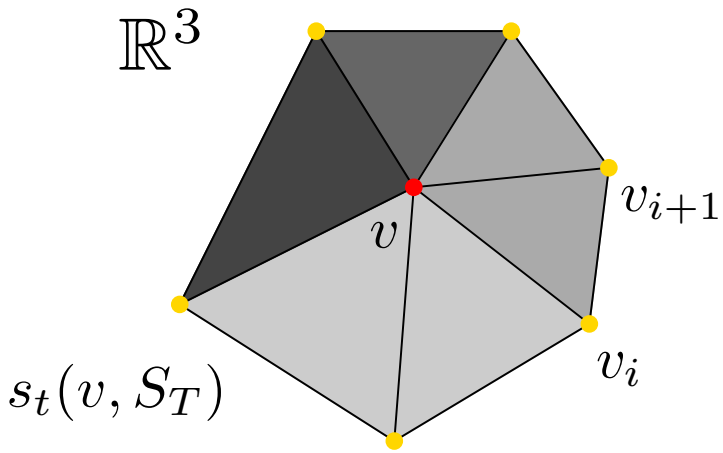
(2) $[v, v_i, v_{i+1}]$ is a triangle of S_T iff $[v', s_v(v'), s_v(v'_{i+1})]$ is a triangle in T_v, where v' is the barycenter of P_v, $i = 0, \ldots, m_v - 1$, and $i + 1$ should be taken congruent modulo m_v.



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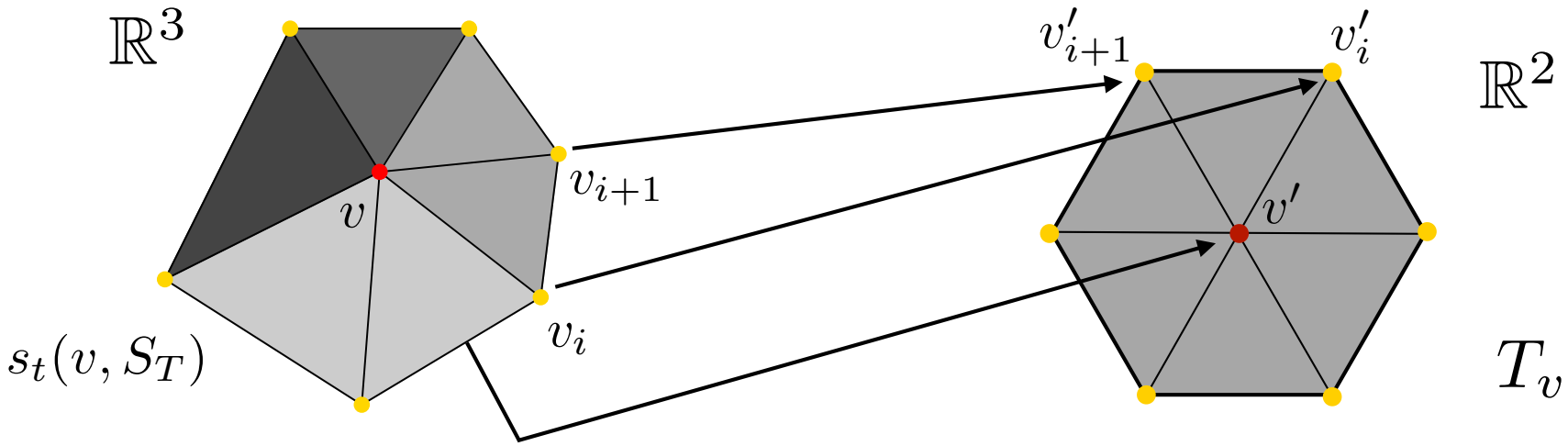

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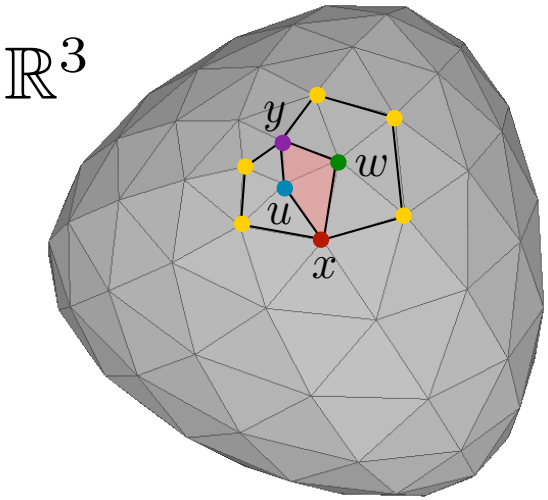
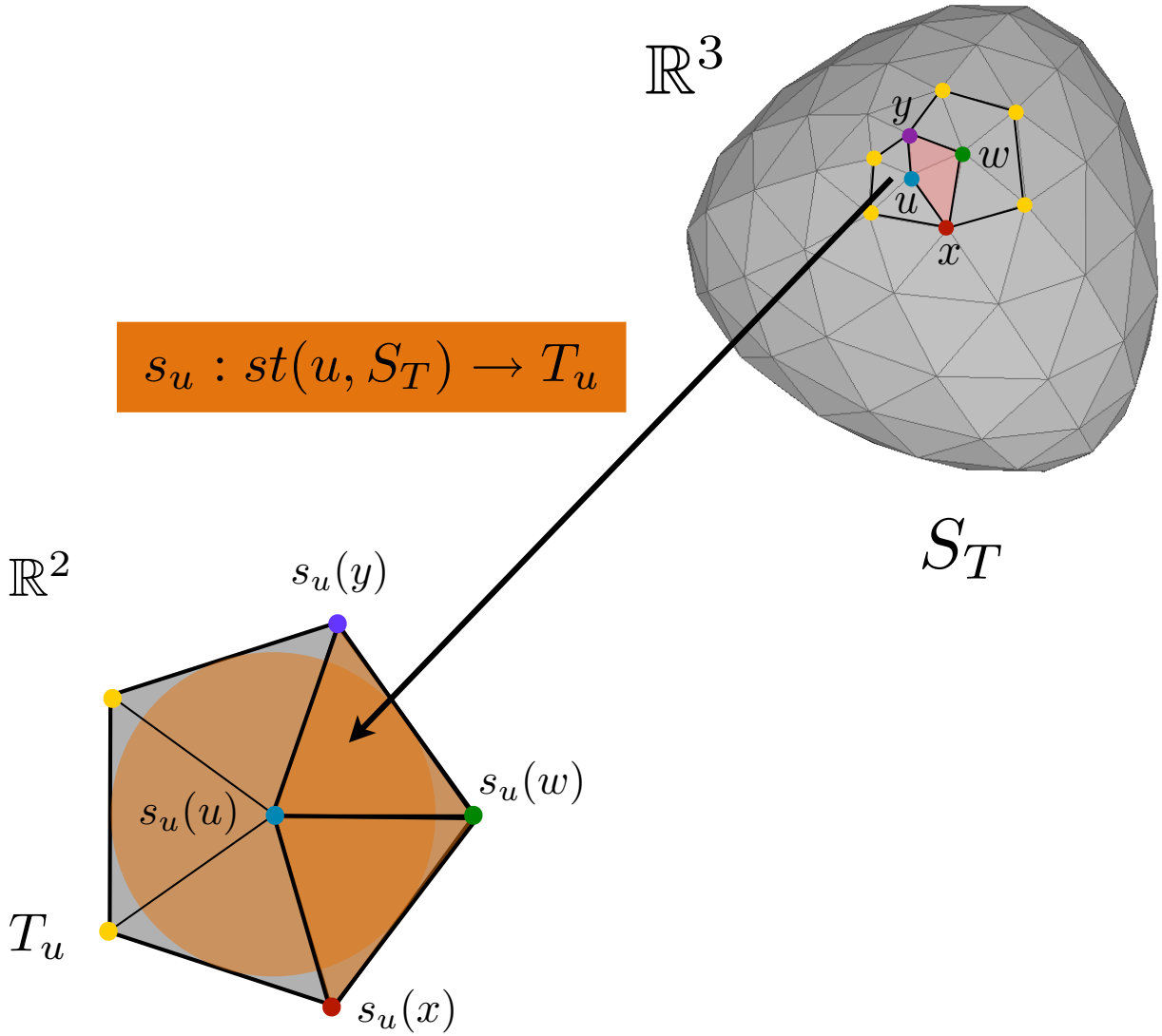


S_T

Building a Set of Gluing Data



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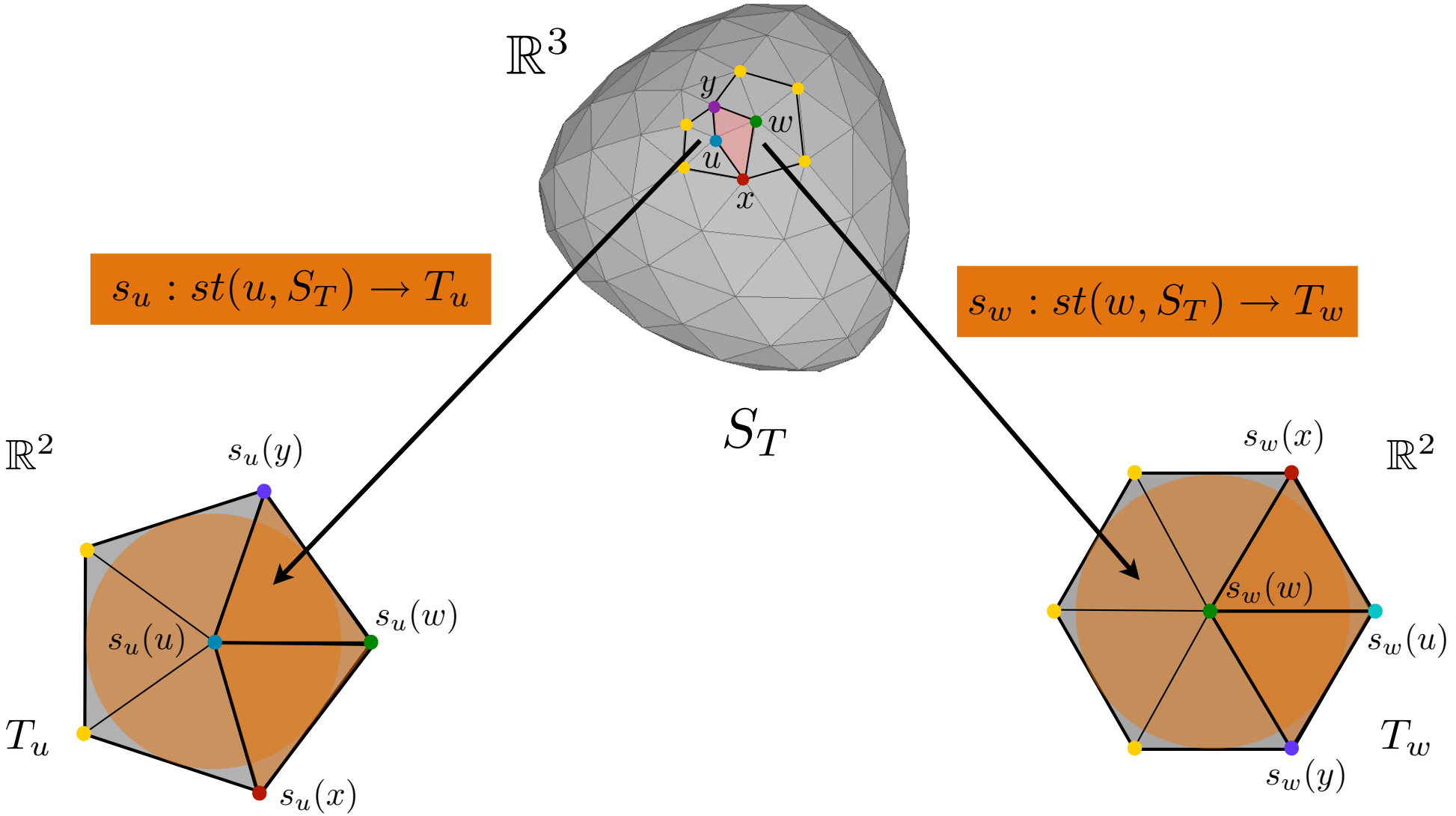

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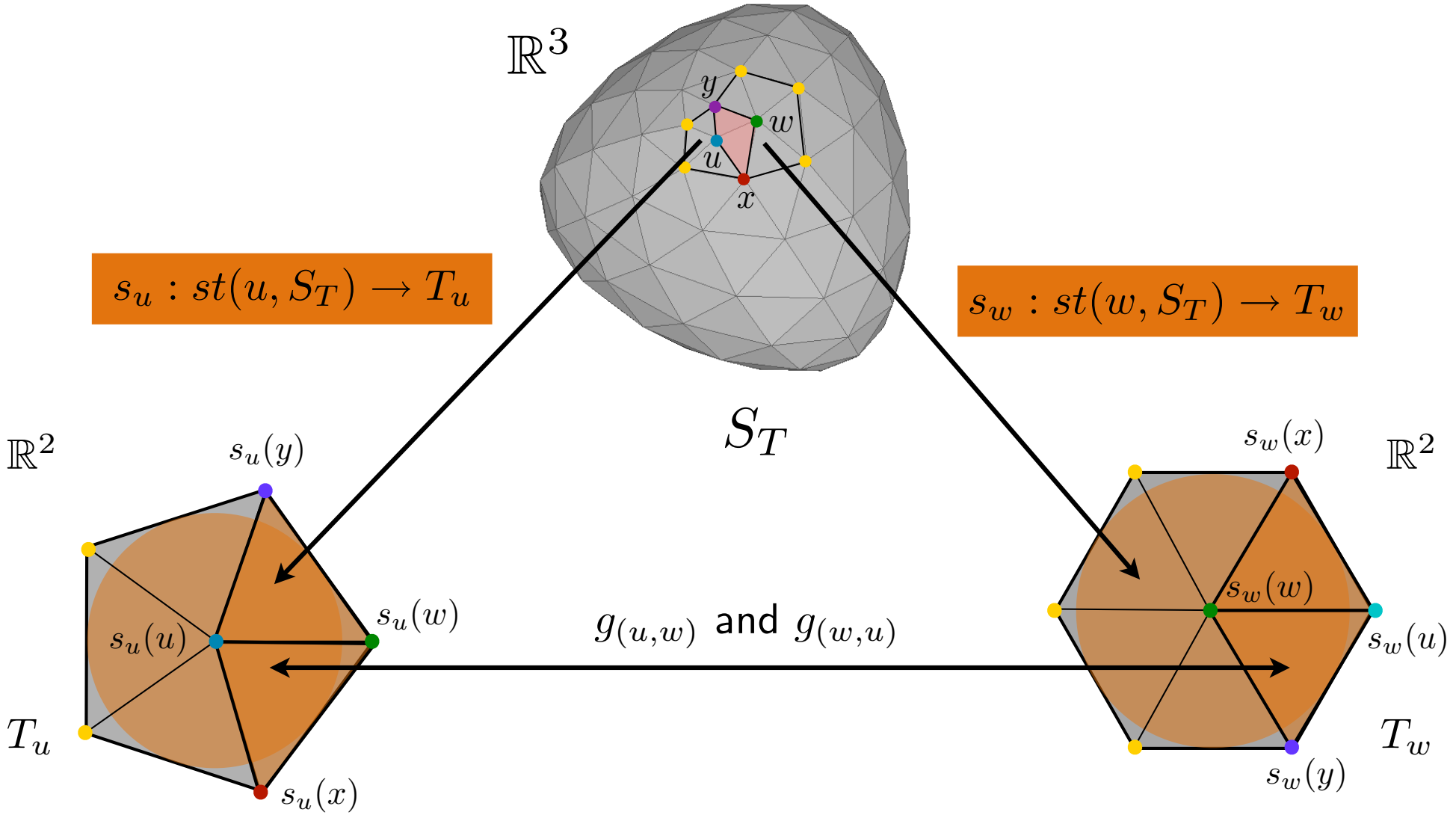

Building a Set of Gluing Data

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For each $u \in I$, let $t_u : \mathbb{R}^2 \to \mathbb{R}^2$ be the unique rigid transformation (i.e., a translation) that takes $(2 \cdot l(u), 0)$ to the origin, $(0, 0)$, of $\mathbb{R}^2$.

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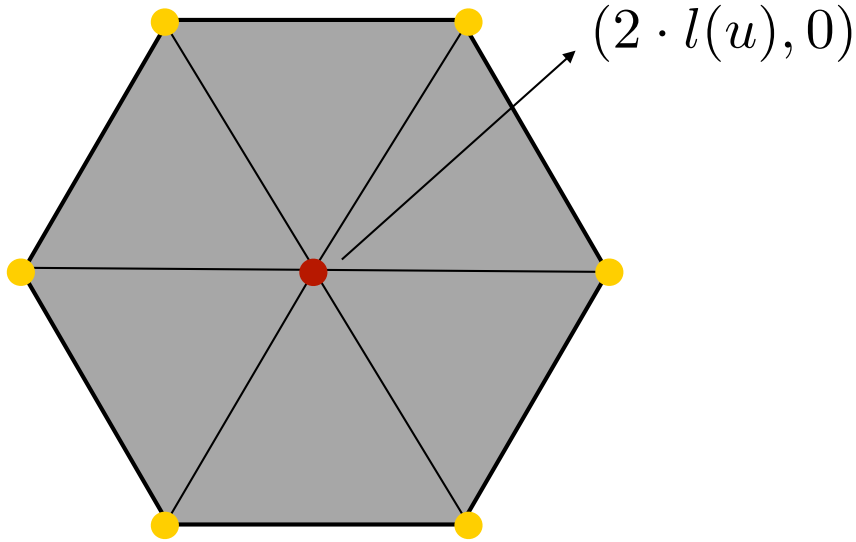
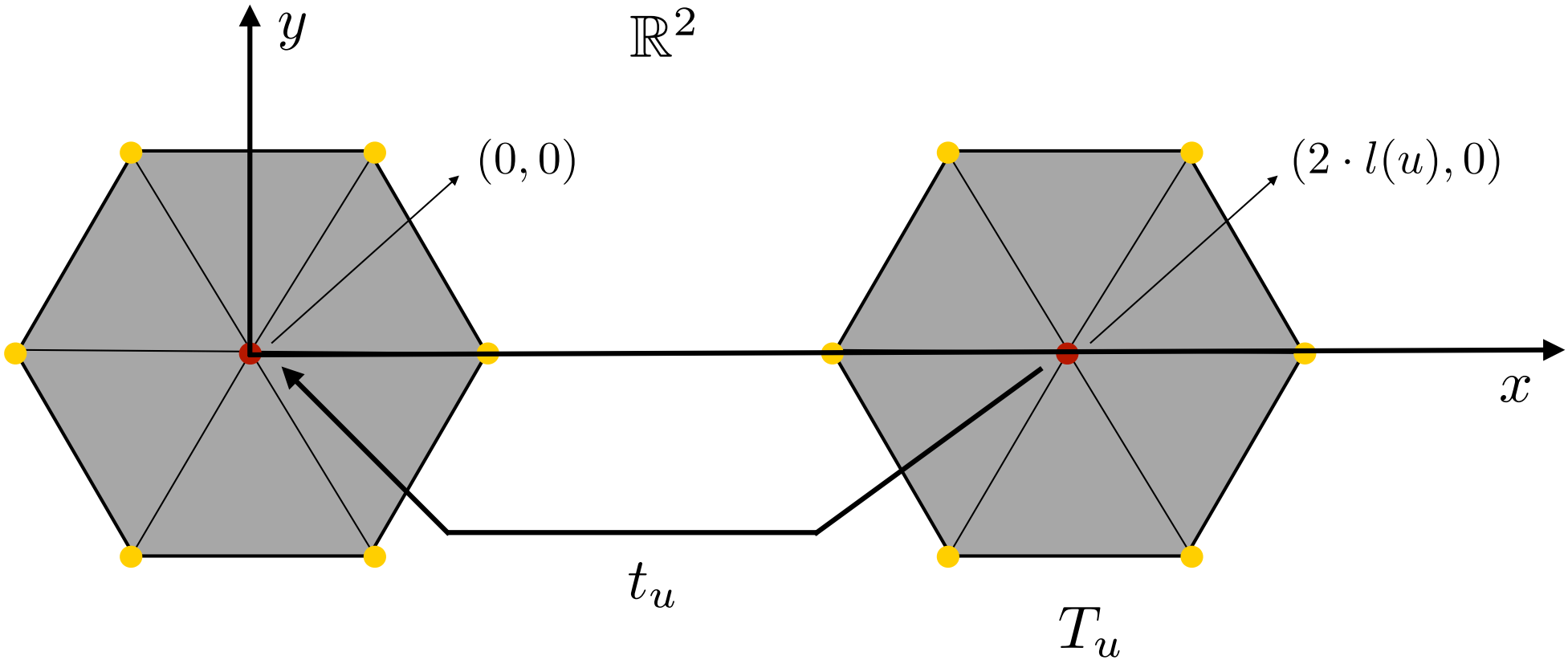


T_u

Building a Set of Gluing Data

For each $u \in I$, let $t_u : \mathbb{R}^2 \to \mathbb{R}^2$ be the unique rigid transformation (i.e., a translation) that takes $(2 \cdot l(u), 0)$ to the origin, $(0, 0)$, of $\mathbb{R}^2$.



Building a Set of Gluing Data

Building a Set of Gluing Data

Let $\Pi : \mathbb{R}^2 - \{(0,0)\} \to (-\pi, \pi] \times \mathbb{R}_+$ be the maps that converts Cartesian to polar coordinates and is given by

$$\Pi(q) = \Pi((x,y)) = (\theta, r)\,,$$

for every $q \in \mathbb{R}^2 - \{(0,0)\}$, where $\theta \in (-\pi, \pi]$ is the **angle** uniquely determined by

$$\cos(x/r) \quad \text{and} \quad \sin(y/r)\,,$$

and $r \in \mathbb{R}_+$ is the **length**, with

$$r = \sqrt{x^2 + y^2}\,.$$

Building a Set of Gluing Data

Building a Set of Gluing Data

Note that Π is bijective and its inverse,

$$\Pi^{-1} : (-\pi, \pi] \times \mathbb{R}_+ \to \mathbb{R}^2 - \{(0,0)\},$$

is given by

$$\Pi^{-1}((\theta, r)) = (r \cdot \cos(\theta), r \cdot \sin(\theta)) \ .$$

Building a Set of Gluing Data

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Note that both Π and Π^{-1} are C^∞ functions.

Building a Set of Gluing Data

Building a Set of Gluing Data

For each $u \in I$, let $f_u : (-\pi, \pi] \times \mathbb{R}_+ \to (-\pi, \pi] \times \mathbb{R}_+$ is given by

$$f_u(q) = f_v((\theta, r)) = \left(\frac{m_u}{6} \cdot \theta, \frac{\cos(\pi/6)}{\cos(\pi/m_v)} \cdot r \right) ,$$

where (θ, r) are the polar coordinates of $q \in \mathbb{R}^2 - \{(0, 0)\}$.

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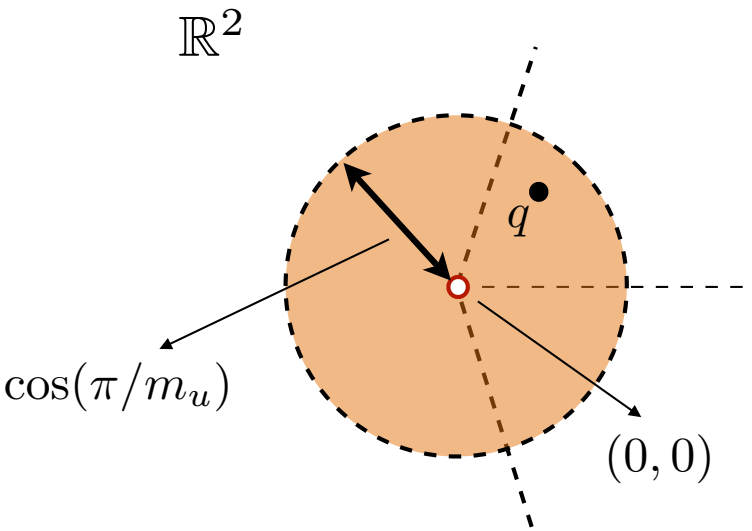

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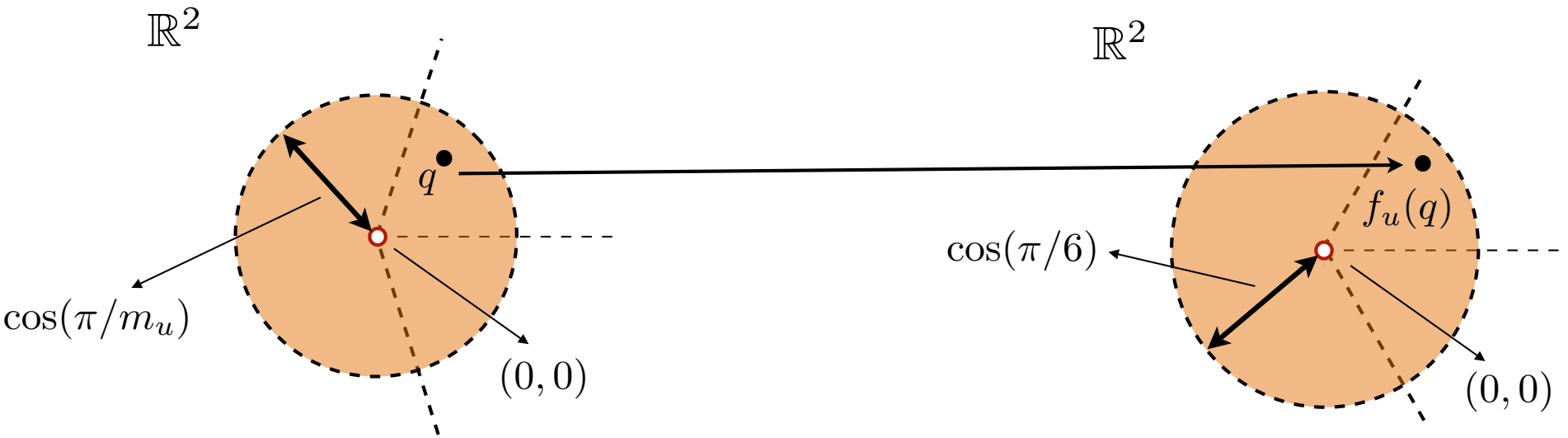

Building a Set of Gluing Data

Building a Set of Gluing Data

For each $u \in I$, let $g_u : \mathbb{R}^2 - \{(2 \cdot l(u), 0)\} \to \mathbb{R}^2 - \{(0,0)\}$ be given by

$$g_u(q) = \Pi^{-1} \circ f_u \circ \Pi \circ t_u(q)\,,$$

for every $q \in \mathbb{R}^2 - \{(2 \cdot l(u), 0)\}$.

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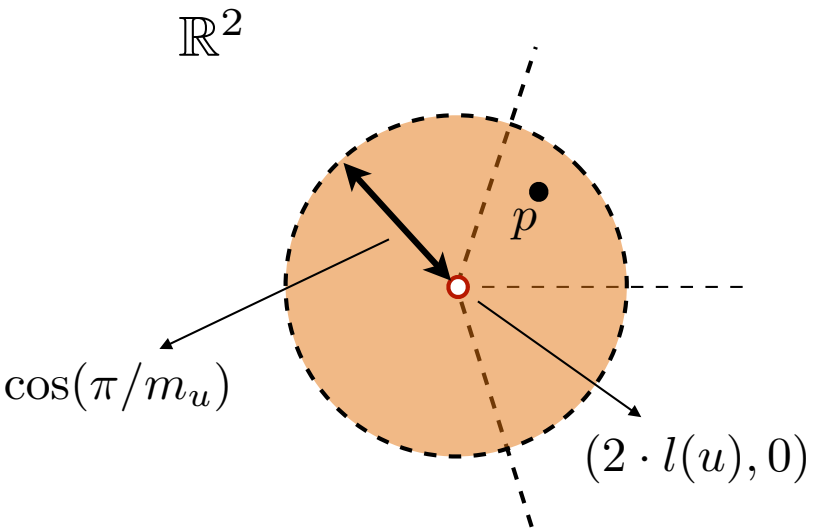

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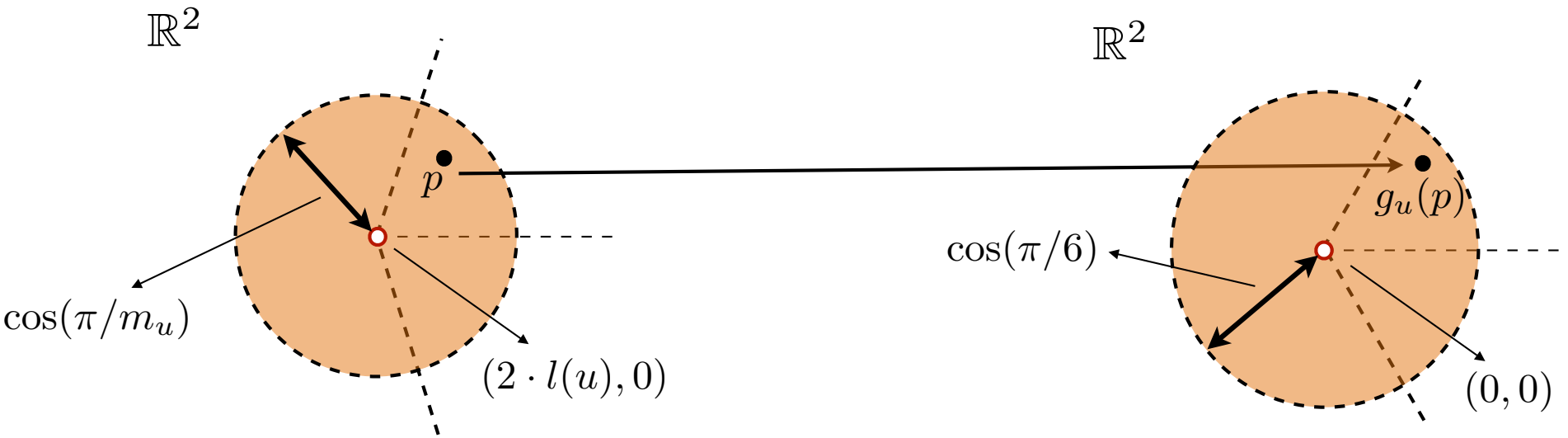

Building a Set of Gluing Data

Building a Set of Gluing Data

Function g_u is bijective and its inverse,

$$g_u^{-1} : \mathbb{R}^2 - \{(0,0)\} \to \mathbb{R}^2 - \{(2 \cdot l(u), 0)\}\,,$$

is given by

$$g_u^{-1}(q) = t_u^{-1} \circ \Pi^{-1} \circ f_u^{-1} \circ \Pi(q)\,,$$

for every $q \in \mathbb{R}^2 - \{(0,0)\}$.

Building a Set of Gluing Data

Building a Set of Gluing Data

Note that

$$f_u^{-1} : (-\pi, \pi] \times \mathbb{R}_+ \to (-\pi, \pi] \times \mathbb{R}_+ \,,$$

the inverse of f_u, is given by

$$f_u^{-1}\left((\beta, s)\right) = \left(\frac{6}{m_u} \cdot \beta, \frac{\cos(\pi/m_u)}{\cos(\pi/6)} \cdot s\right) \,,$$

where (β, s) are the polar coordinates of $q \in (-\pi, \pi] \times \mathbb{R}_+$.

Building a Set of Gluing Data

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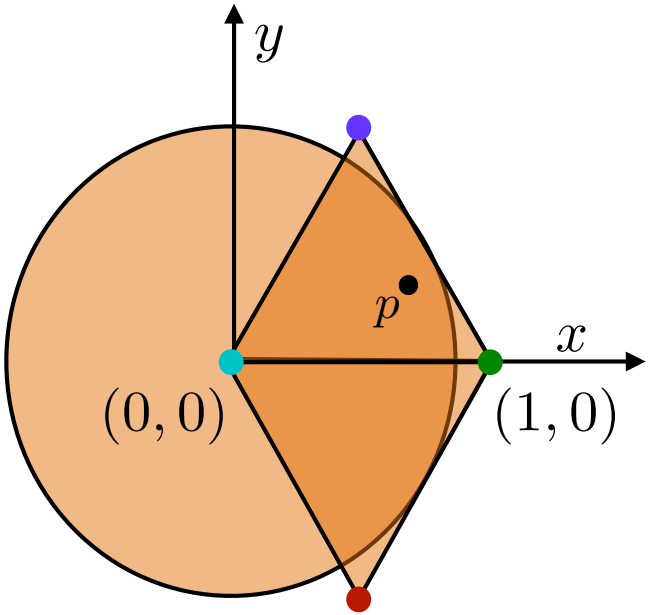

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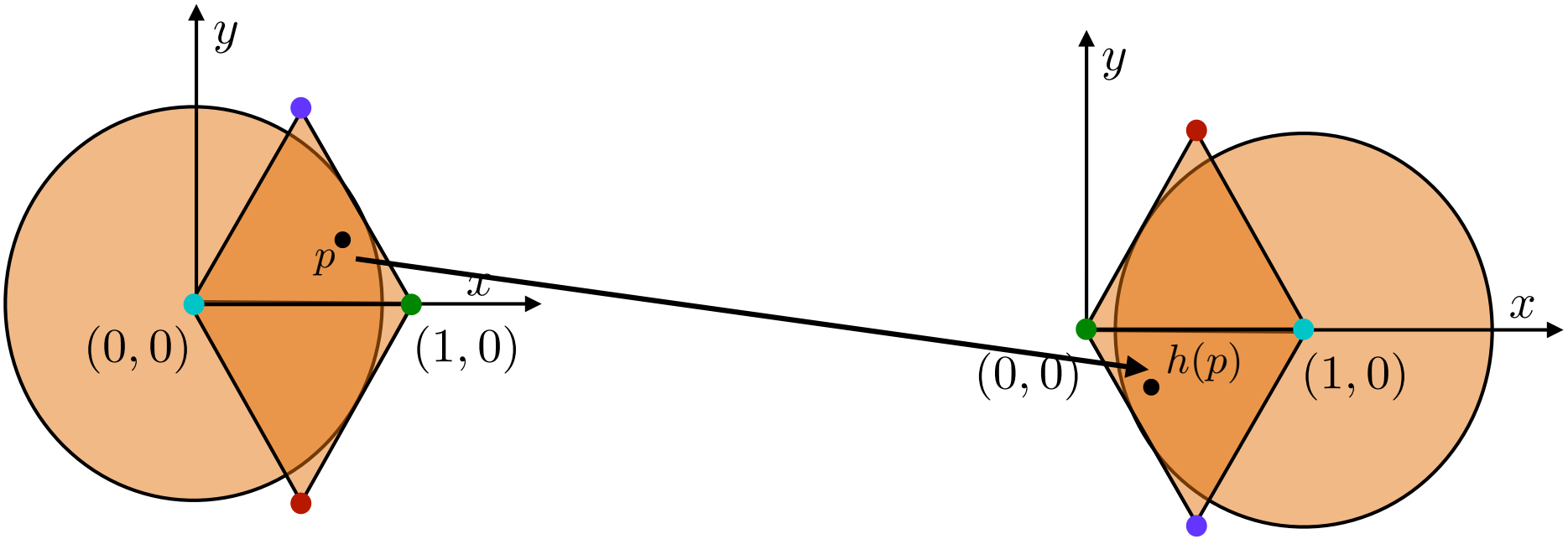

Building a Set of Gluing Data

Building a Set of Gluing Data

Let $[u, w]$ be an edge of S_T and $R_{(u,w)}$ be the rotation around $(2 \cdot l(u), 0)$ that identifies the edge $[s_u(u) = u', s_u(w)]$ of T_u with its edge $[u', u'_0]$.

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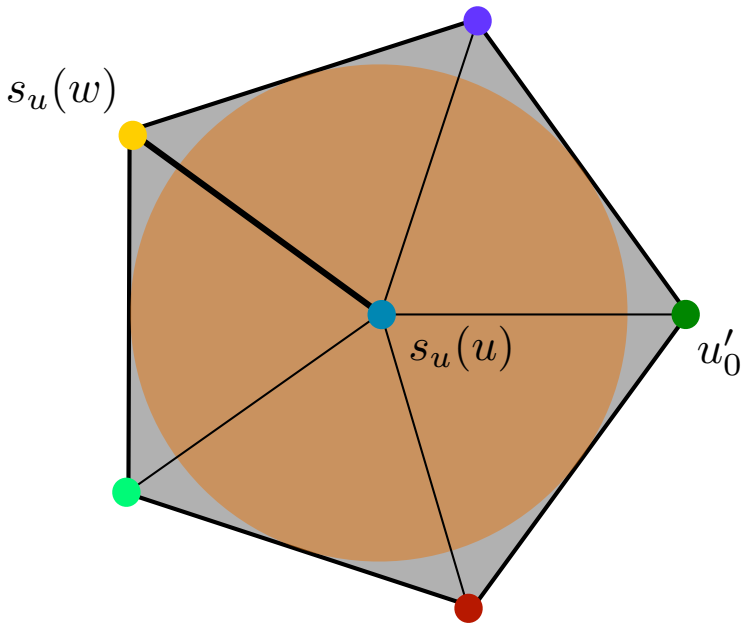

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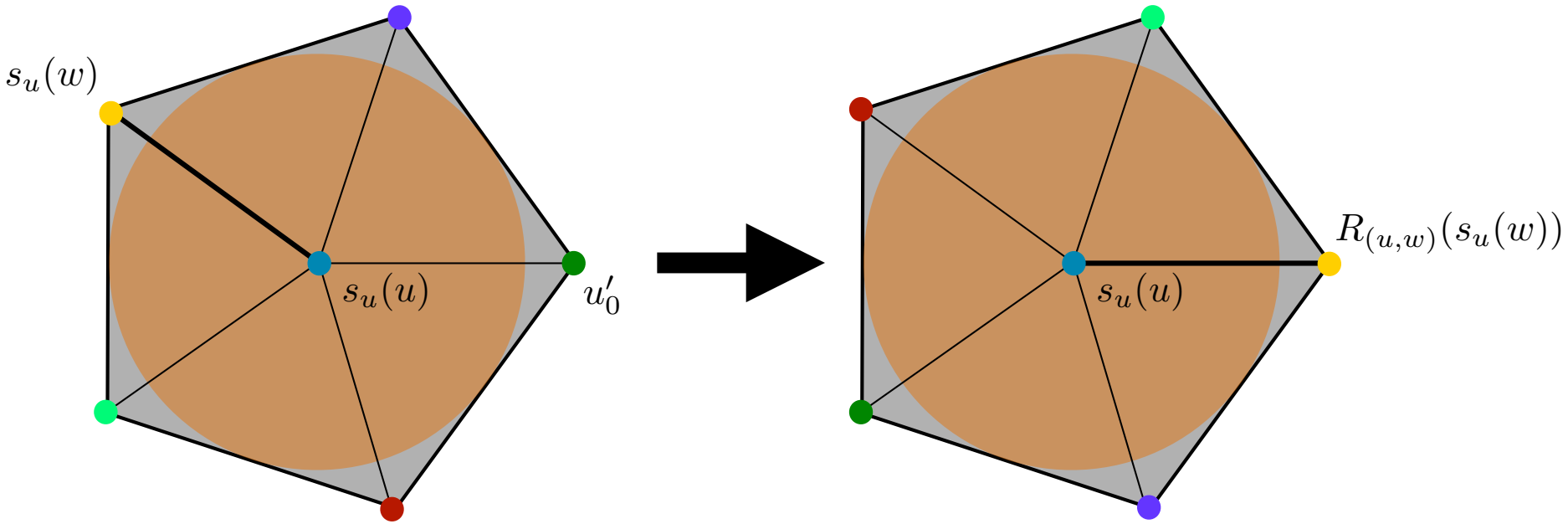

Building a Set of Gluing Data

Building a Set of Gluing Data

Finally, for any two vertices, u and w of S_T, such that $[u, w]$ is an edge of S_T, let

$$g_{(u,w)} : \Omega_u - \{(2 \cdot l(u), 0)\} \to g_{(u,w)}(\Omega_u - \{(2 \cdot l(u), 0)\}),$$

be the composite function given by

$$g_{(u,w)}(p) = R^{-1}_{(w,u)} \circ g_w^{-1} \circ h \circ g_u \circ R_{(u,w)}(p),$$

for every $p \in \Omega_u - \{(2 \cdot l(u), 0)\}$.

Building a Set of Gluing Data

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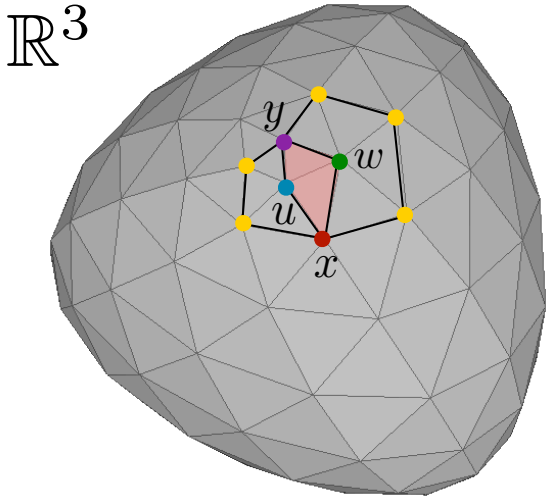

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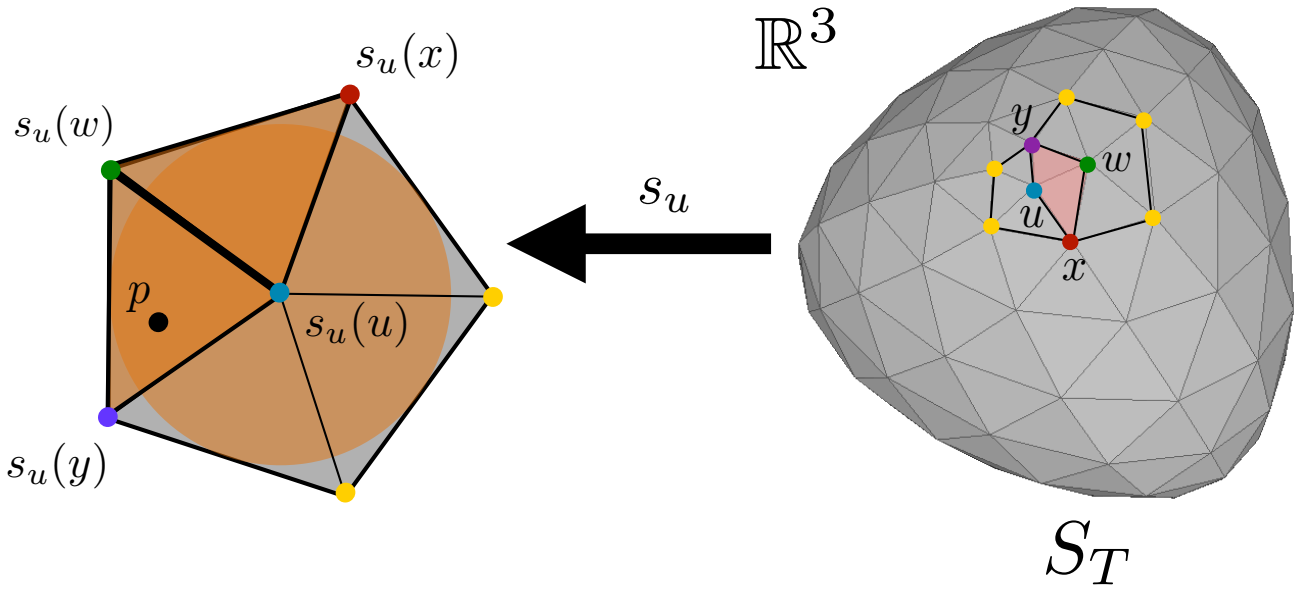

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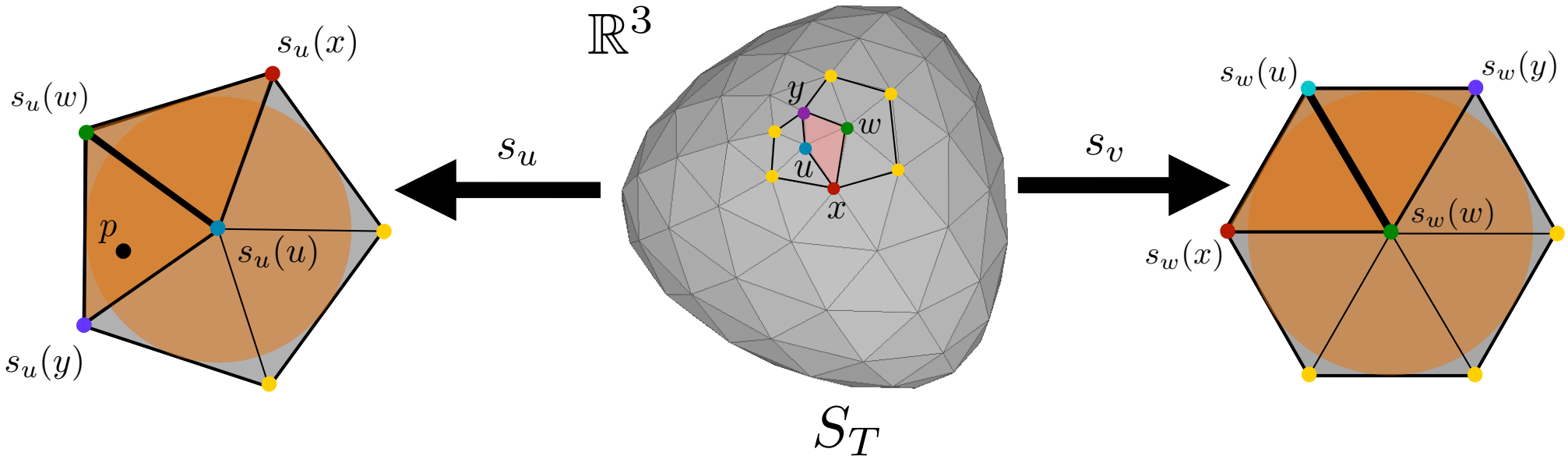

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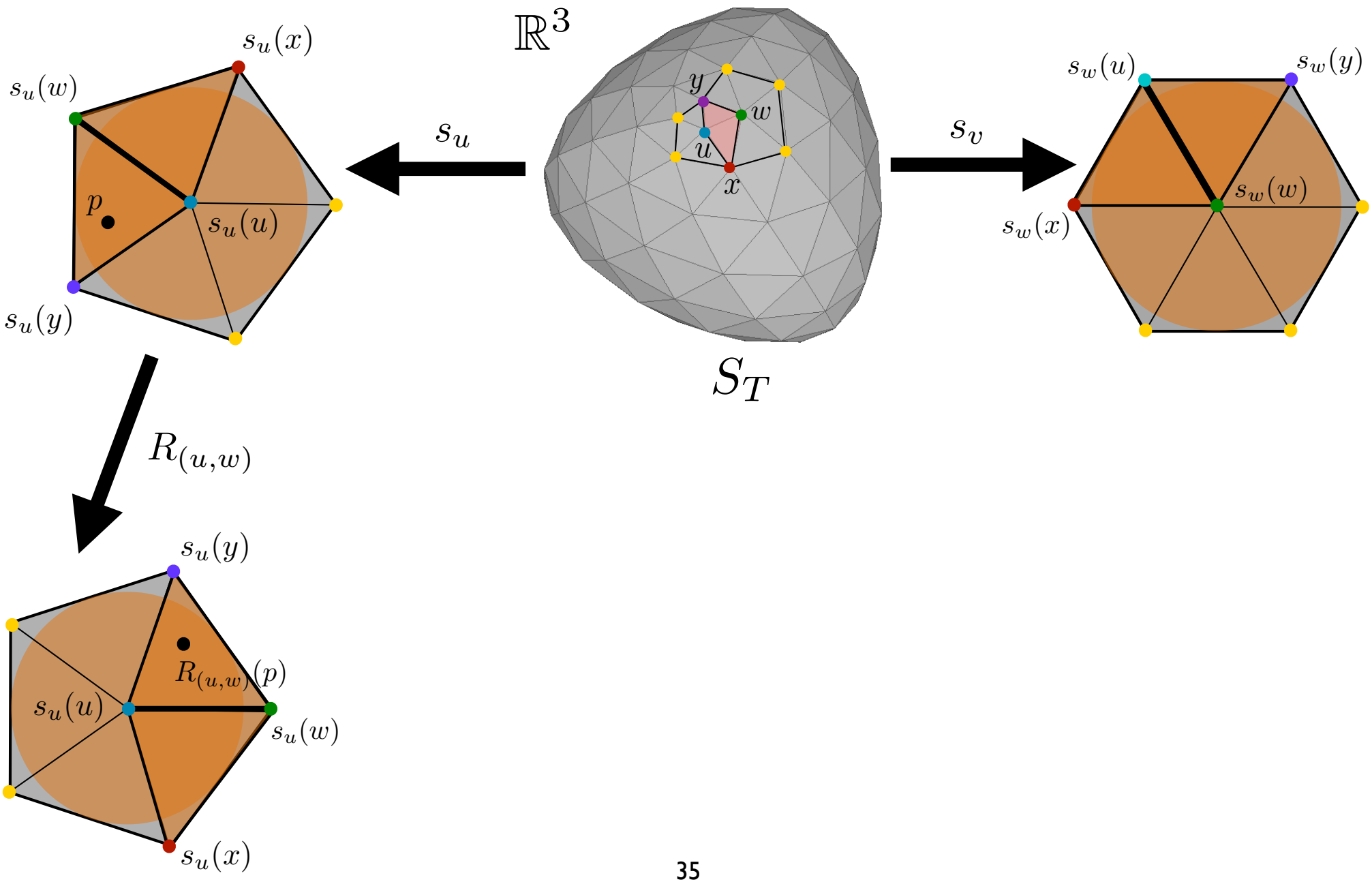

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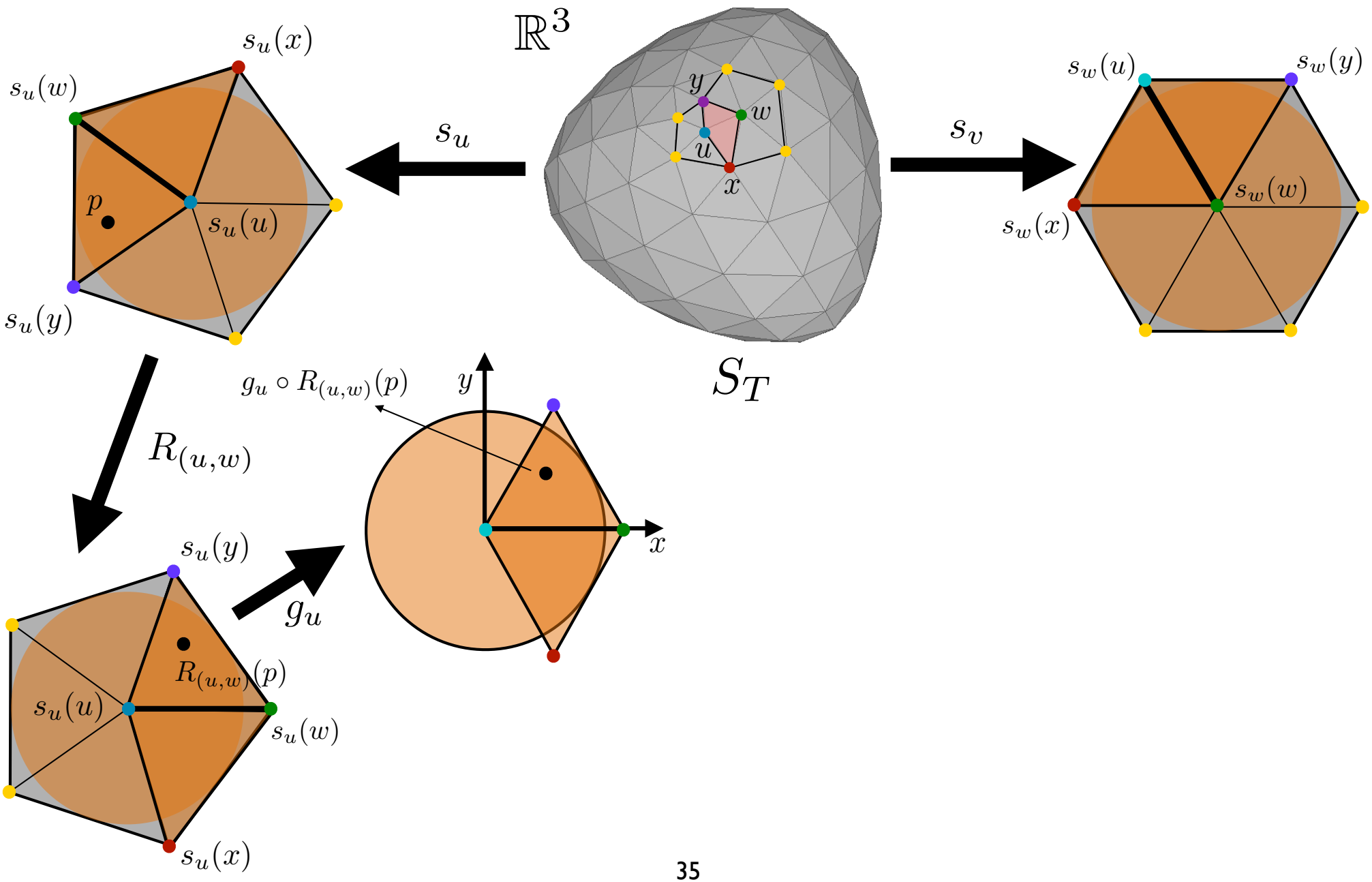

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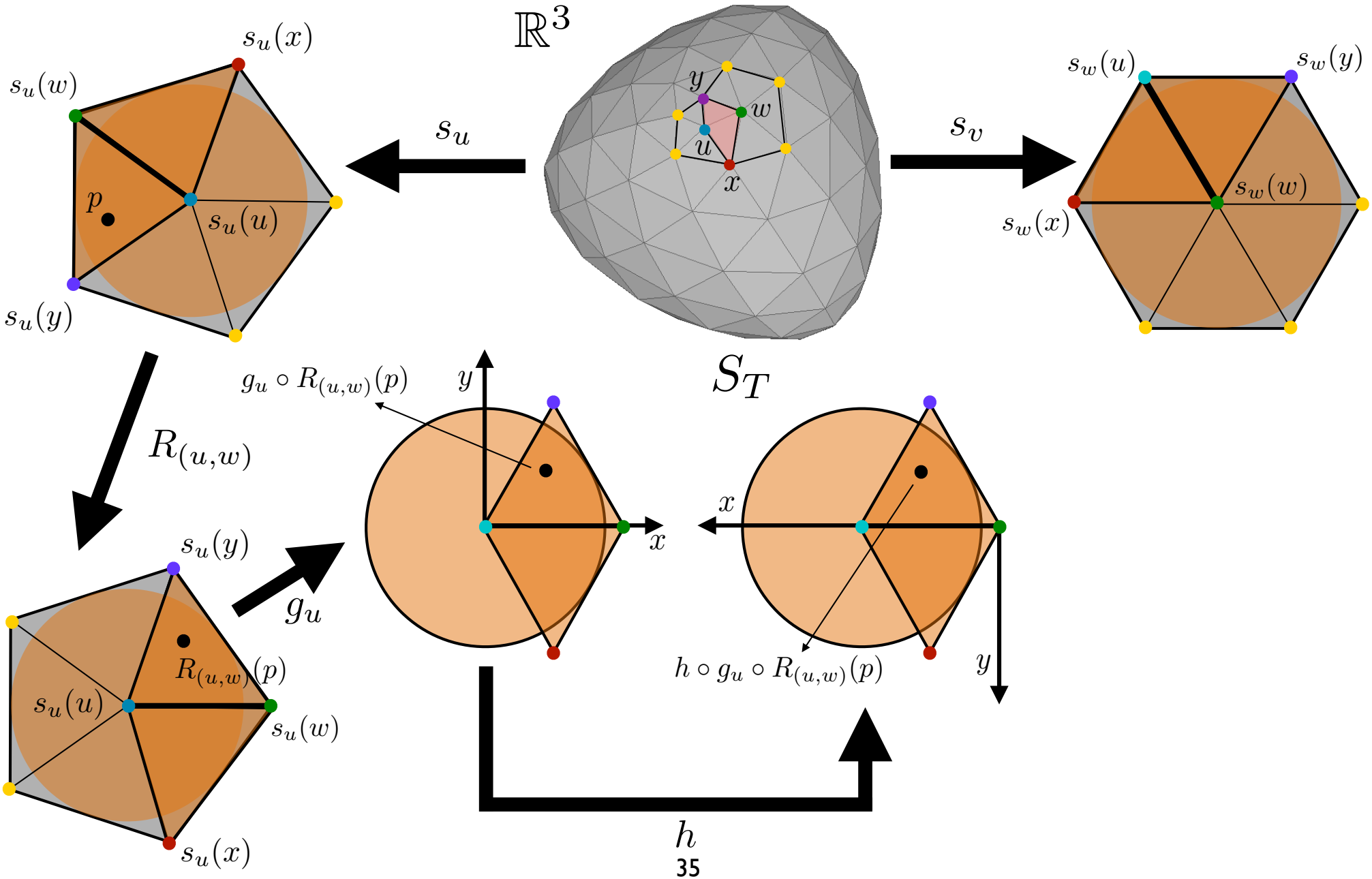

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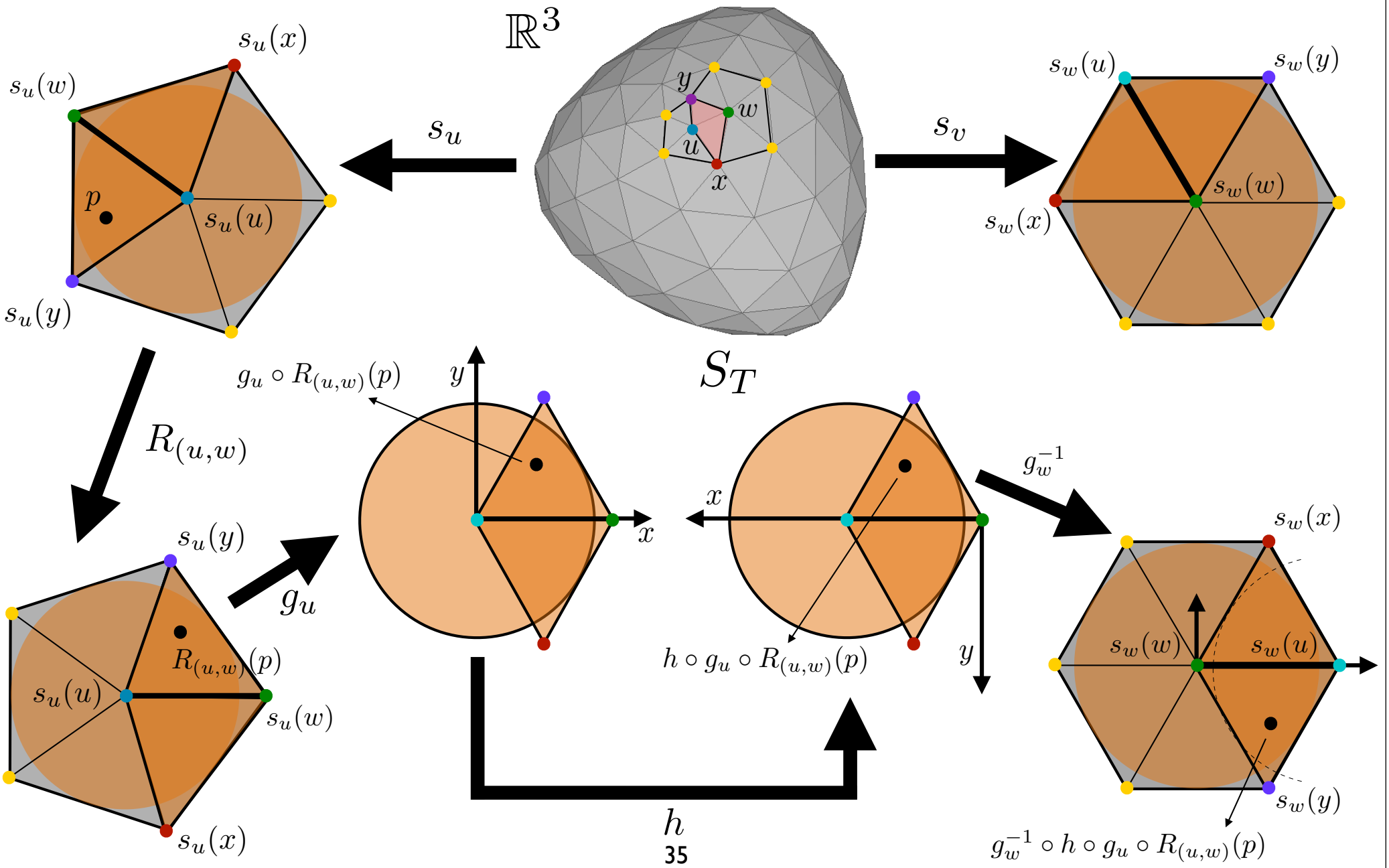

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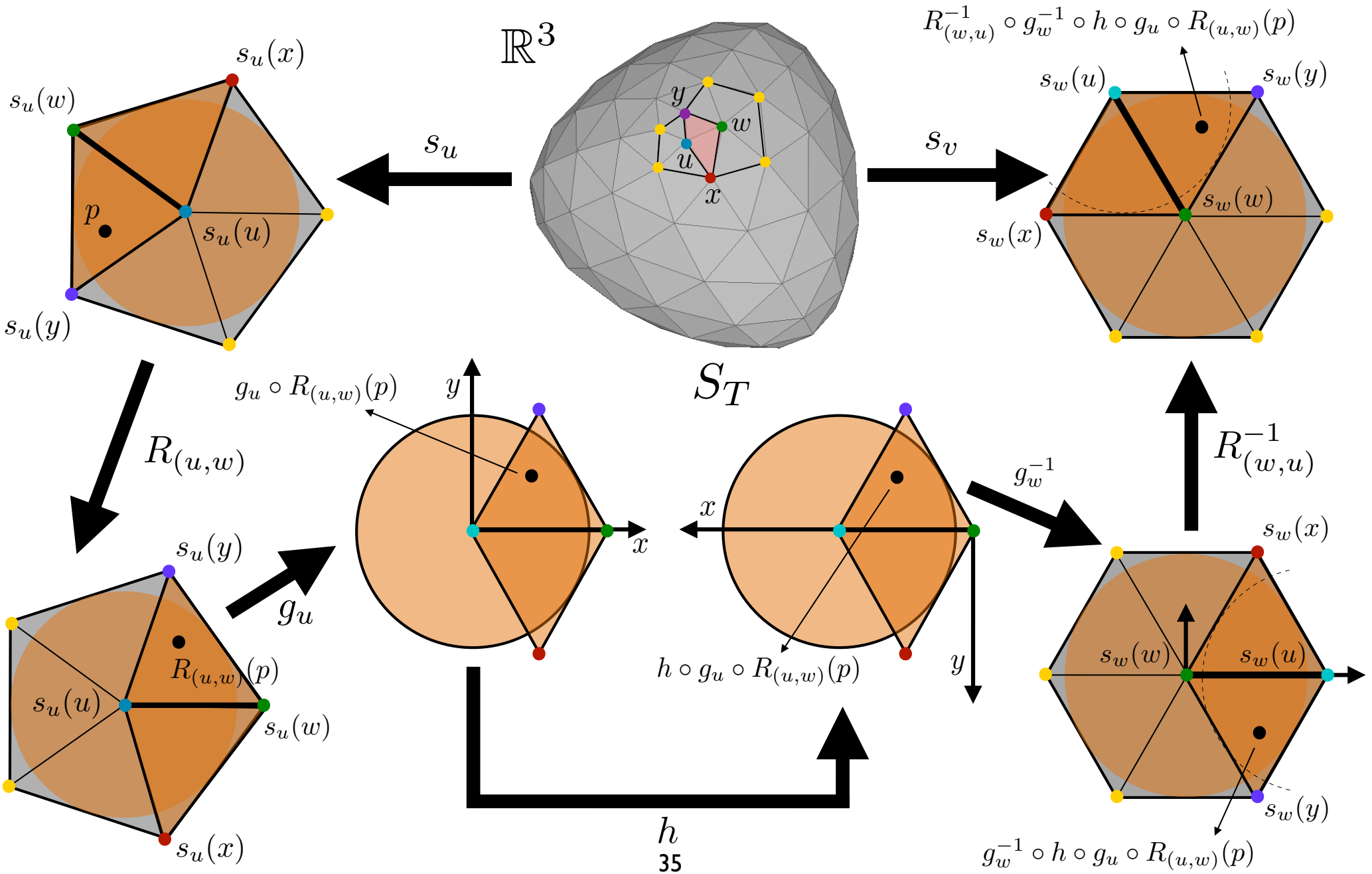

Building a Set of Gluing Data

Building a Set of Gluing Data

For any two vertices, $u, w \in I$, the **gluing domain**, Ω_{uw}, is defined as

$$\Omega_{uw} = \begin{cases} \Omega_u & \text{if } u = w, \\ g_{(w,u)}(\Omega_w - \{(2 \cdot l(w), 0)\}) \cap \Omega_u & \text{if } [u, w] \text{ is an edge of } S_T, \\ \emptyset & \text{otherwise.} \end{cases}$$

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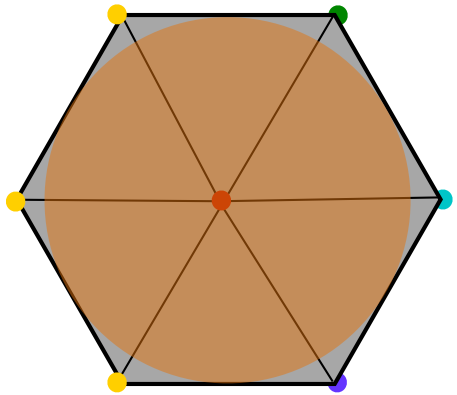
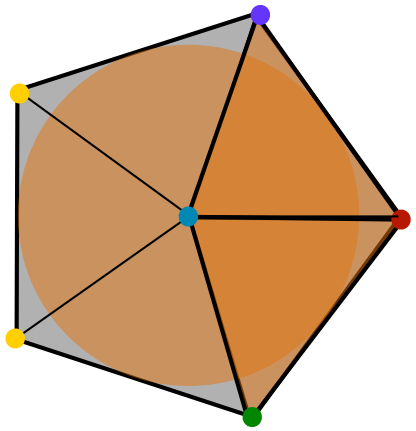


T_u



T_w

Building a Set of Gluing Data

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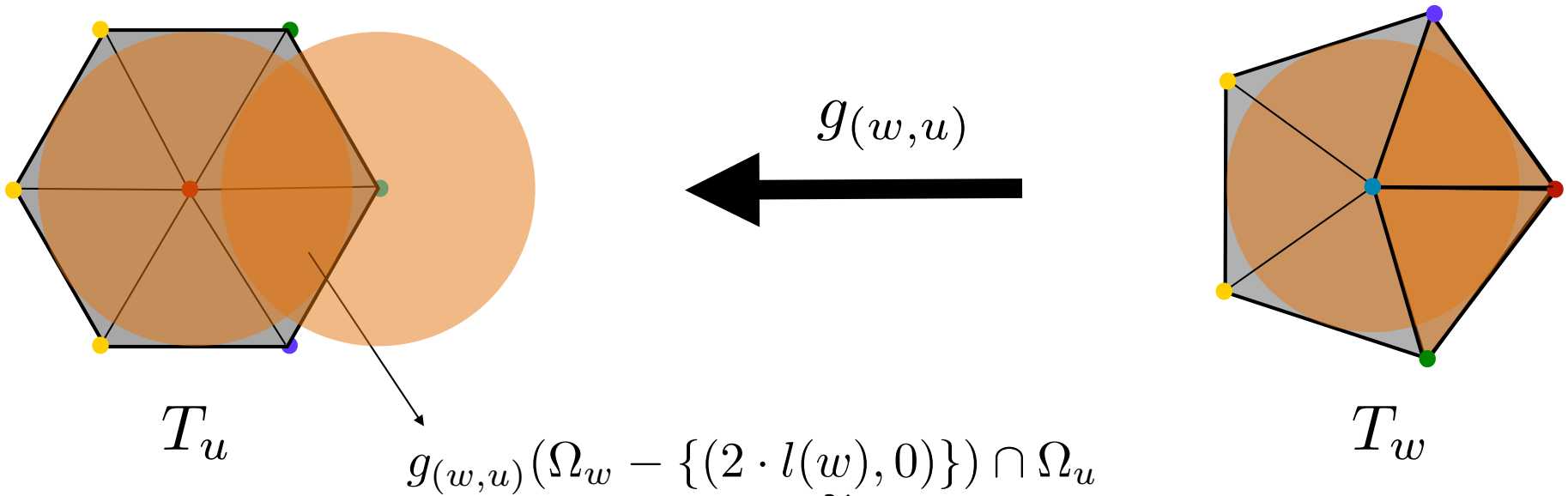

Building a Set of Gluing Data

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We can show that the above definition of gluing domain satisfies condition (2) of the definition of sets of gluing data we saw before:

Building a Set of Gluing Data

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(2) For every pair $(i, j) \in I \times I$, the set Ω_{ij} is an open subset of Ω_i. Furthermore, $\Omega_{ii} = \Omega_i$ and $\Omega_{ji} \neq \emptyset$ if and only if $\Omega_{ij} \neq \emptyset$.

Building a Set of Gluing Data

Building a Set of Gluing Data

Transition functions

Building a Set of Gluing Data

Transition functions

Let

$$K = \{(u, w) \in I \times I \mid \Omega_{uw} \neq \emptyset\}\,.$$

Building a Set of Gluing Data

Building a Set of Gluing Data

For any pair, $(u, w) \in K$, we define $\varphi_{wu} : \Omega_{uw} \to \Omega_{wu}$, the **transition function from Ω_u to Ω_w**, by the following expression

$$\varphi_{wu}(p) = \begin{cases} p & \text{if } u = w, \\ g_{(w,u)}(p) & \text{otherwise} \end{cases}$$

for every point $p \in \Omega_{uw}$.

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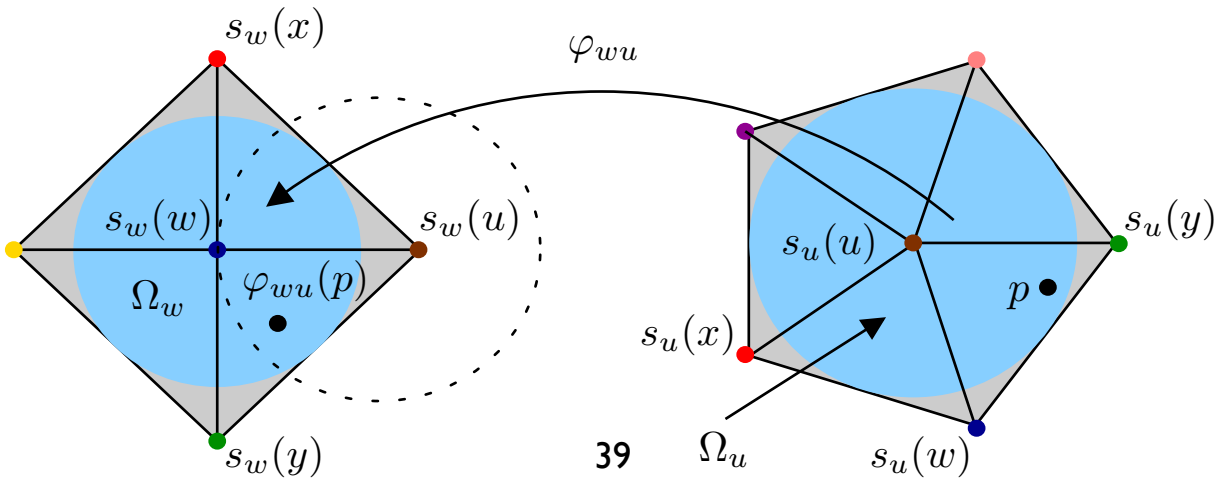

Building a Set of Gluing Data

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(a) $\varphi_{ii} = \mathrm{id}_{\Omega_i}$, for all $i \in I$,

(b) $\varphi_{ij} = \varphi_{ji}^{-1}$, for all $(i, j) \in K$, and

(c) for all i, j, and k, if $\Omega_{ji} \cap \Omega_{jk} \neq \emptyset$ then $\varphi_{ji}^{-1}(\Omega_{ji} \cap \Omega_{jk}) \subseteq \Omega_{ik}$ and $\varphi_{ki}(x) = \varphi_{kj} \circ \varphi_{ji}(x)$, for all $x \in \varphi_{ji}^{-1}(\Omega_{ji} \cap \Omega_{jk})$.

Conclusions

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We learned *one* way of defining a set of gluing data from a triangle mesh.

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The set of gluing data has n_v p-domains, and $2 \cdot n_e$ non-empty gluing domains and transition functions, where n_v and n_e are the number of vertices and edges, respectively, of the input triangle mesh.

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The set of gluing data has n_v p-domains, and $2 \cdot n_e$ non-empty gluing domains and transition functions, where n_v and n_e are the number of vertices and edges, respectively, of the input triangle mesh.

Our transition functions are C^∞ (which is a very desirable property), but they are not *very* simple (i.e., linear, affine, polynomial).

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What can we say about our construction?

Conclusions

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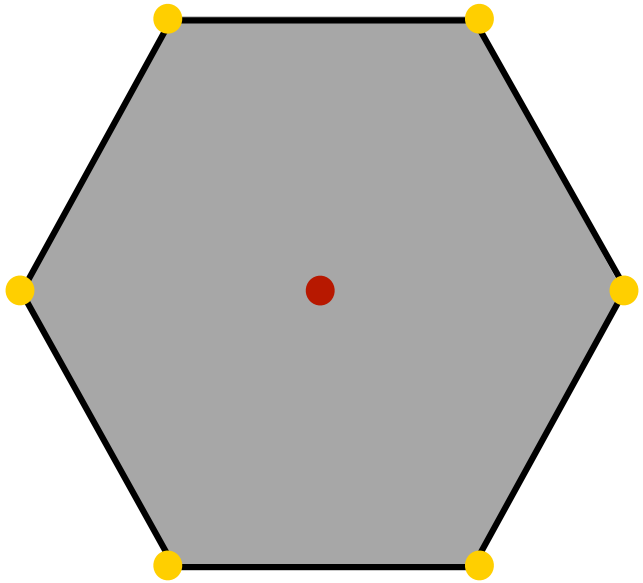
Why didn't we let the interior of the P-polygons be the p-domains?



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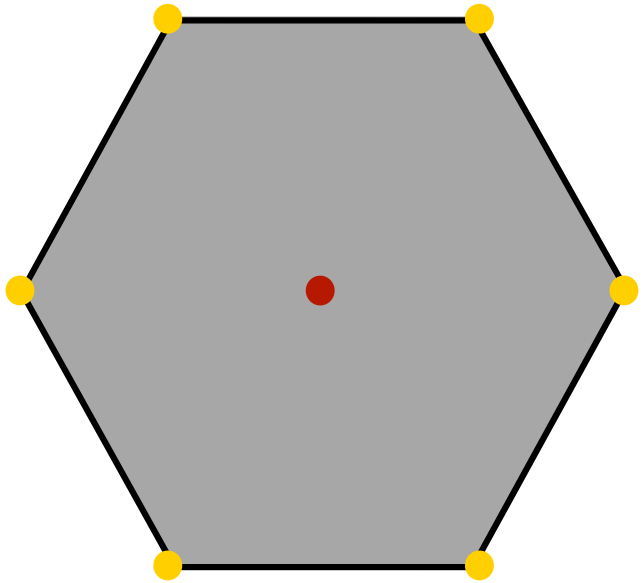

Simple answer: we failed to figure out the transition maps!

Conclusions

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OPEN PROBLEM: Can you find a **simple** C^∞ bijective map g satisfying $g_{(v,w)} = g_{(u,w)} \circ g_{(v,u)}$ (this has to do with the cocycle condition)?

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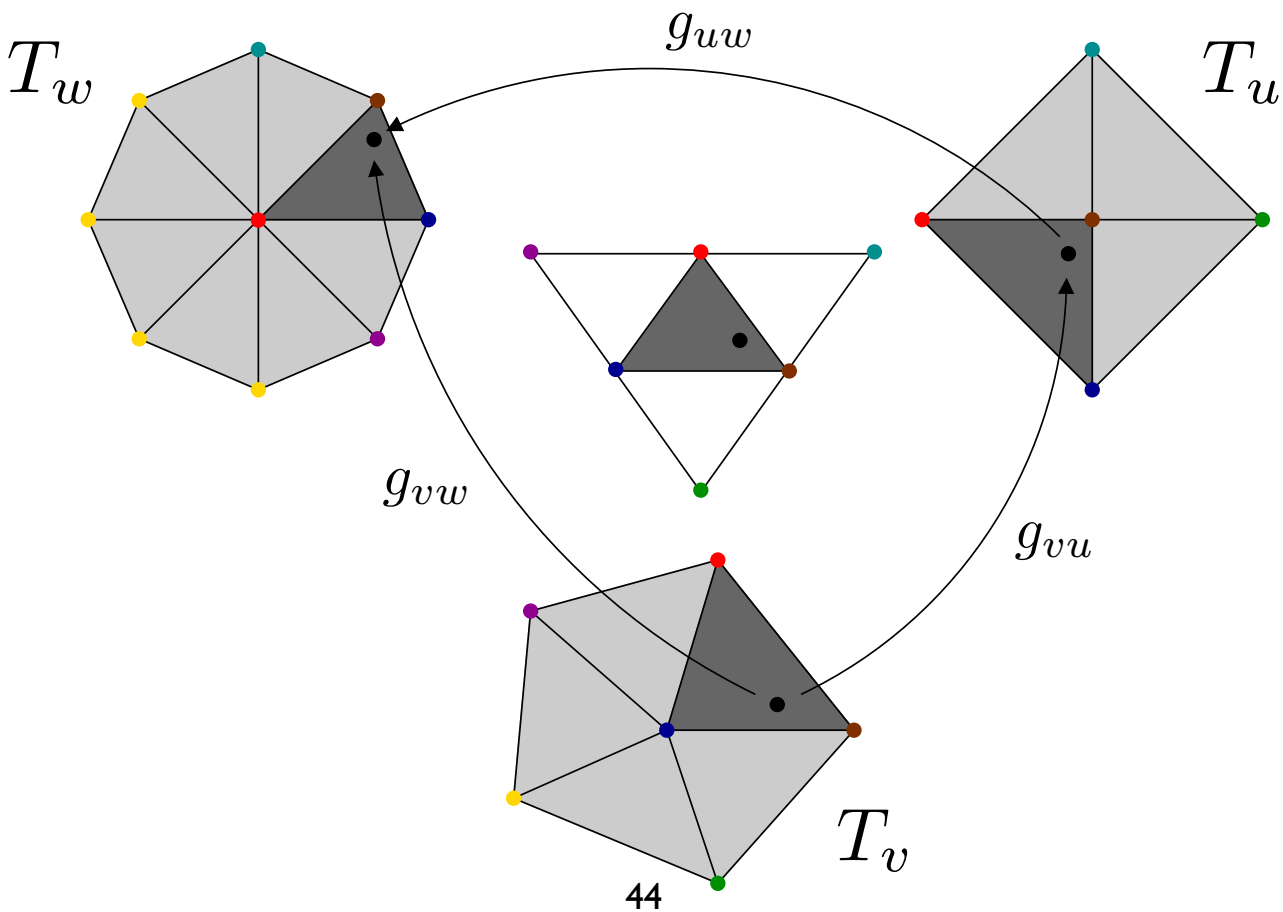

Suggested Reading

Suggested Reading

- Siqueira, M;, Xu, D;, Gallier, J.; Morera, D;, Nonato, L. G.; Velho, L.; *A New Construction of Smooth Surfaces from Triangle Meshes Using Parametric Pseudo-Manifolds*, preprint submitted to Computer & Graphics.

  Download a PDF from the course web page:

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