Implementation Details

Lecture 7 - February 6, 2009 - 1-3 PM

Outline

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- Data Structure
- Algorithms
- Conclusions
- Suggested Readings
- Acknowledgments

Data Structure

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We can use any "decent" existing data structure for surface meshes.

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Basic elements:

Data Structure

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Basic elements:

vertex

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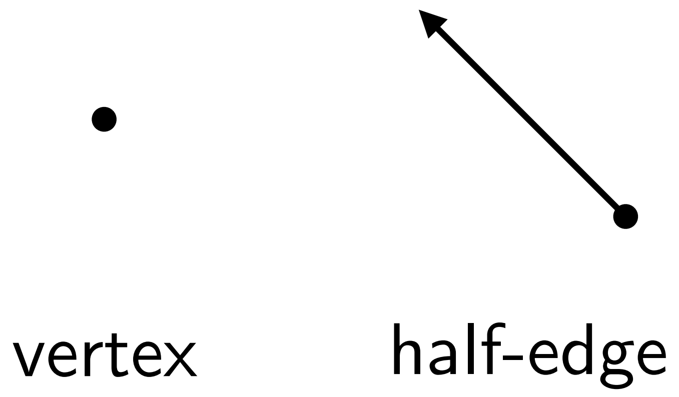

Data Structure

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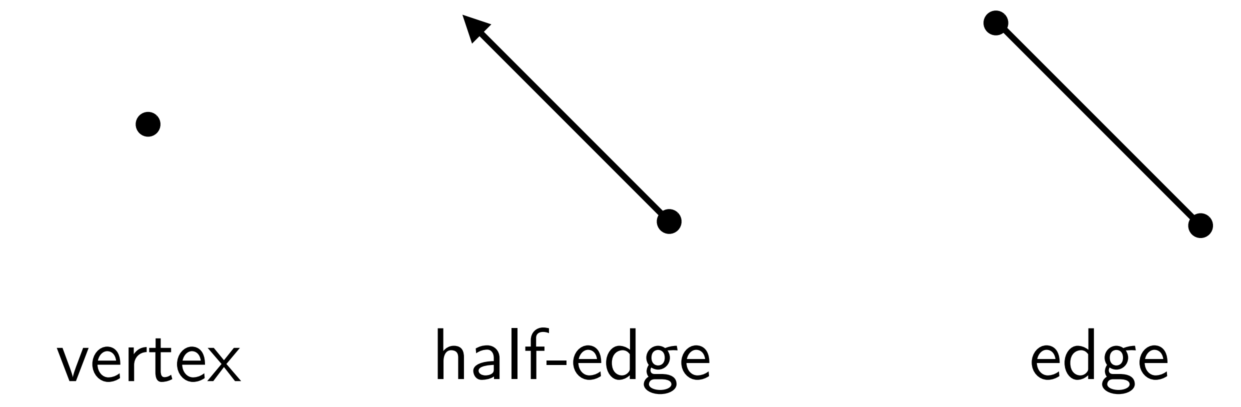

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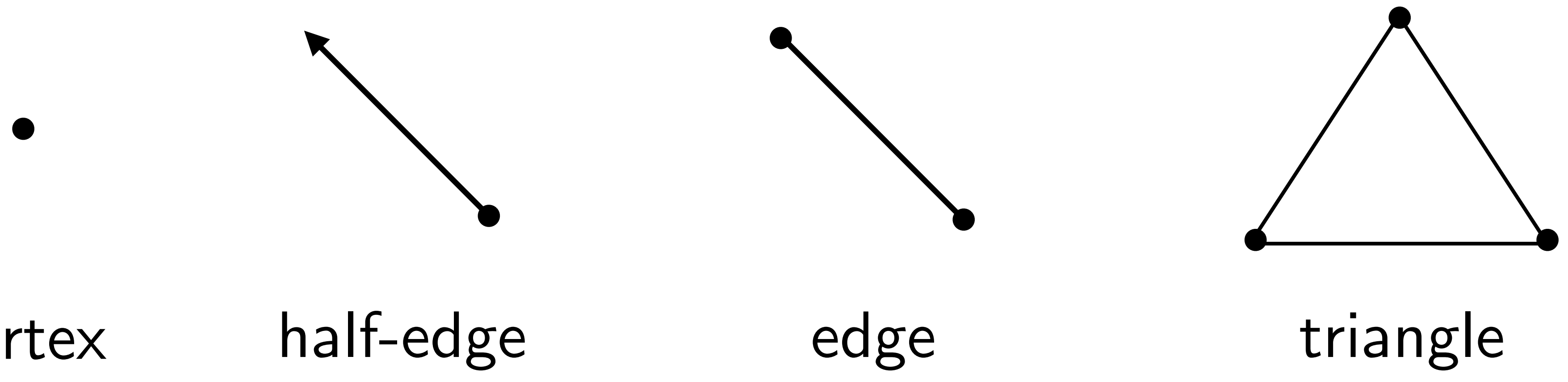


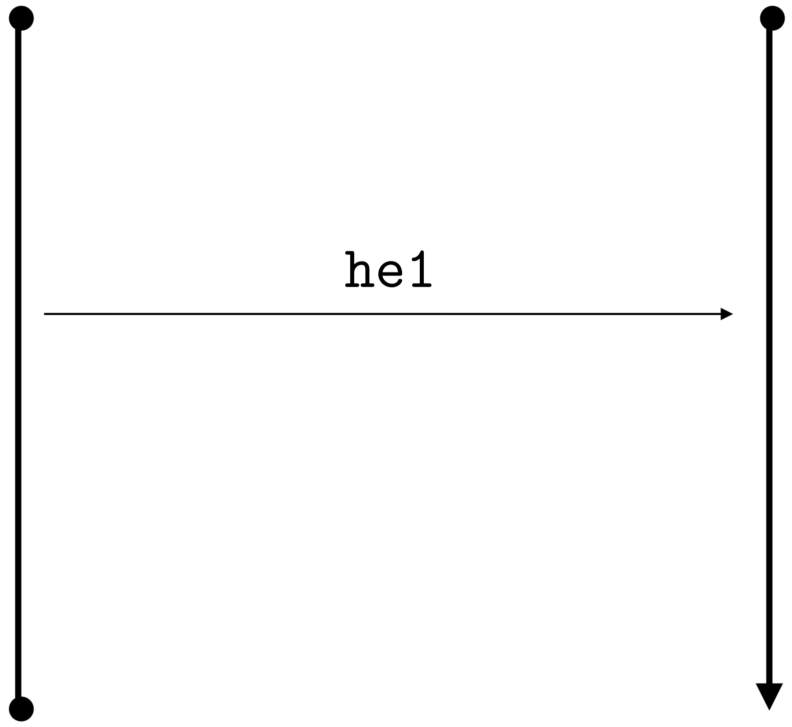
Data Structure

Data Structure

- Edge

Data Structure

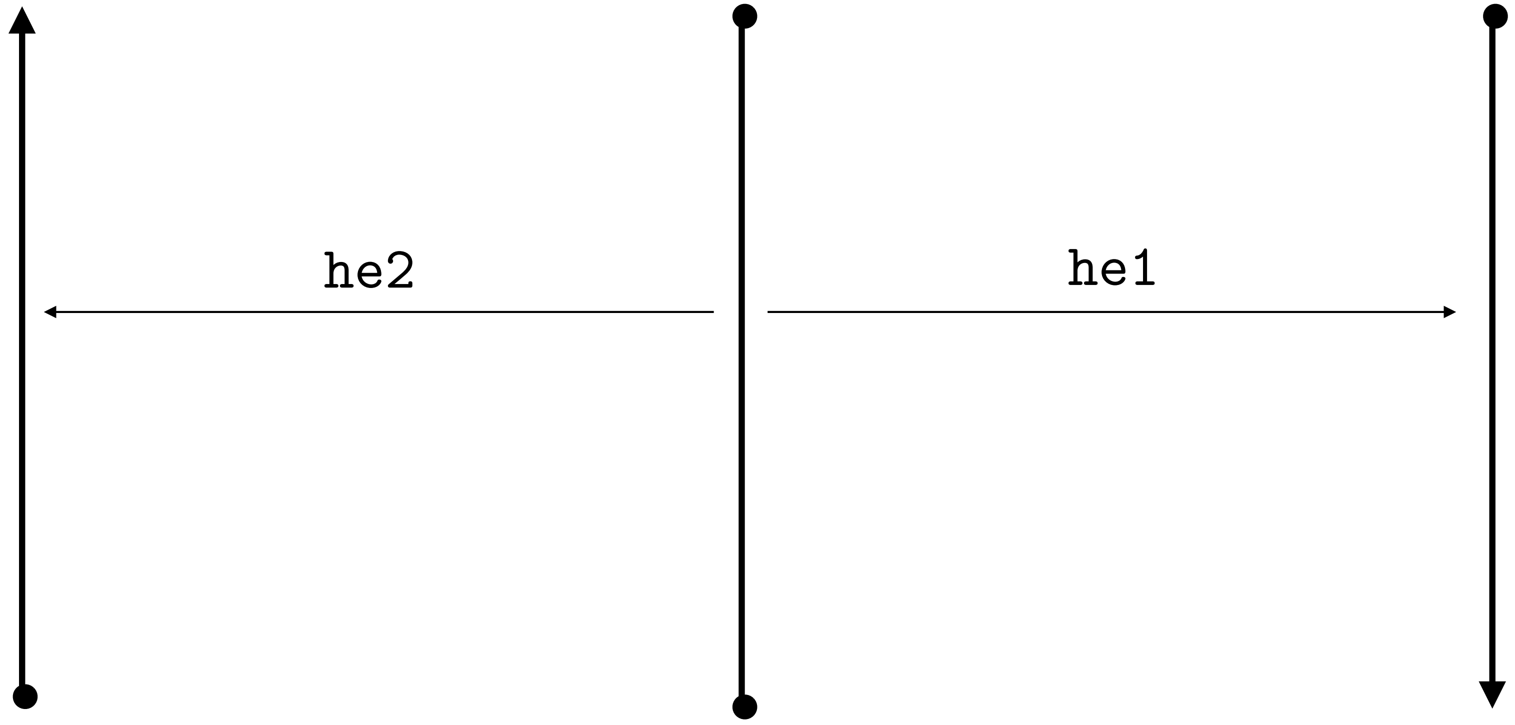
- Edge



Pointer to half-edge 1

Data Structure

- Edge

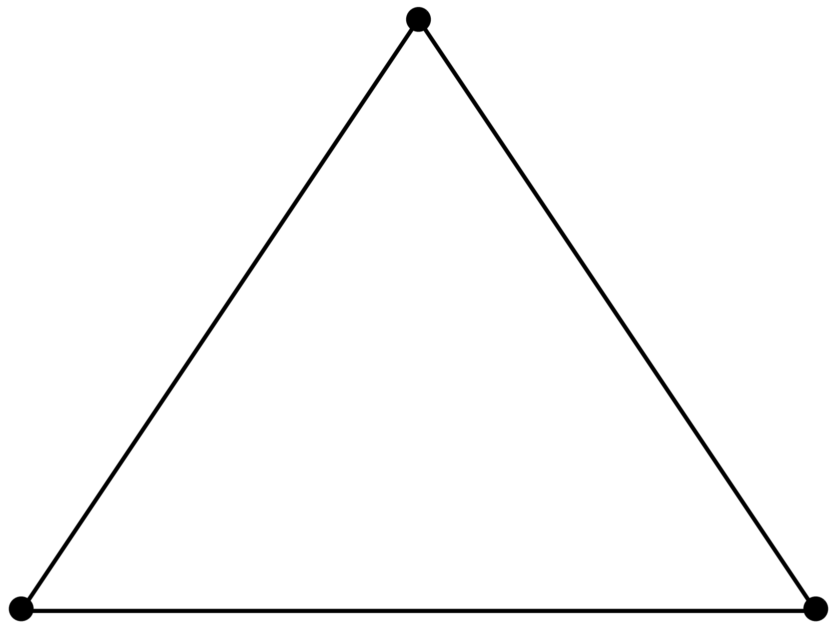


Pointer to half-edge 2 Pointer to half-edge 1

Data Structure

Data Structure

- Triangle



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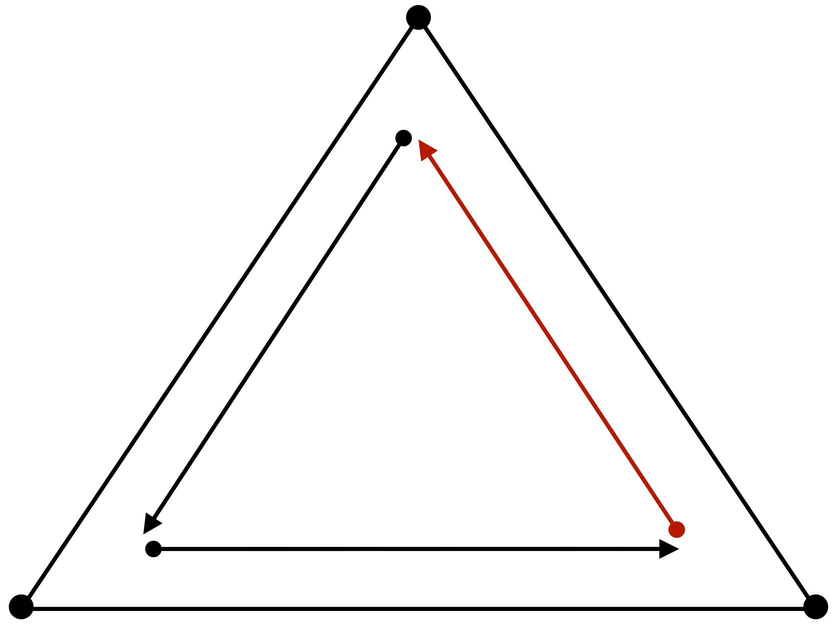

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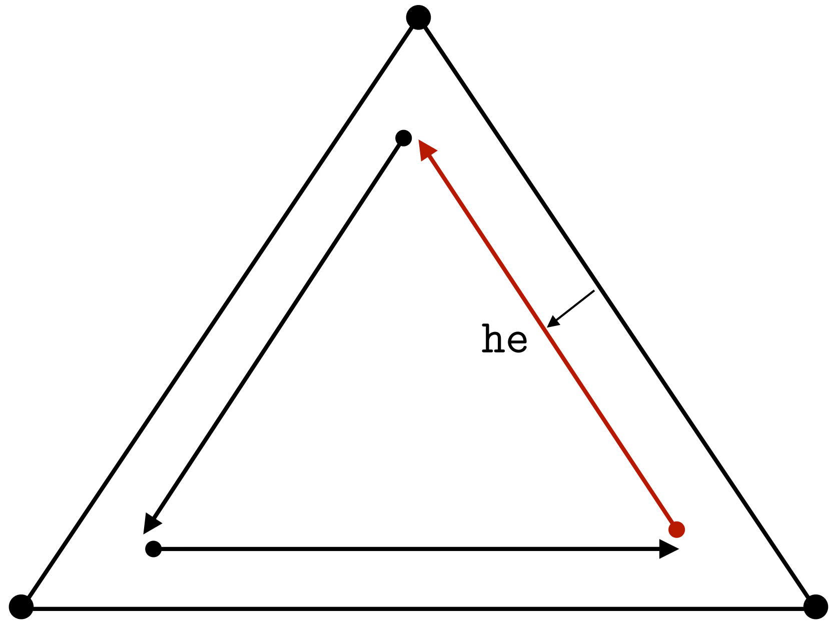

Pointer to one half-edge (known as the *first*)

Data Structure

Data Structure

- Half-edge

Data Structure

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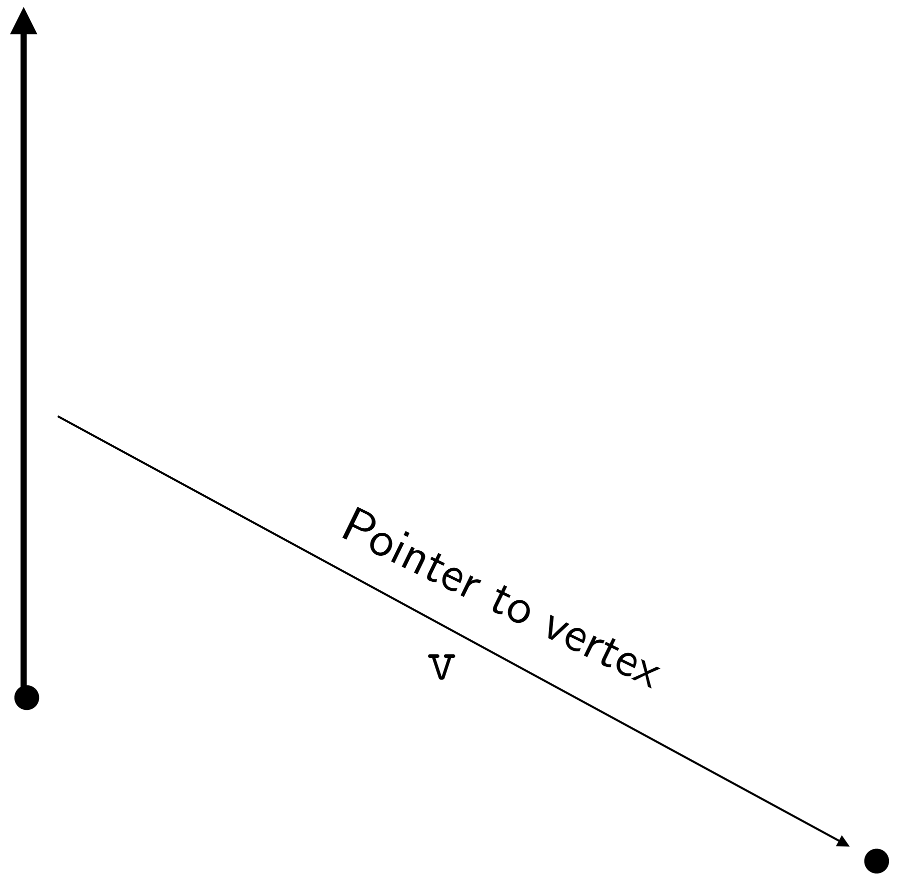

Data Structure

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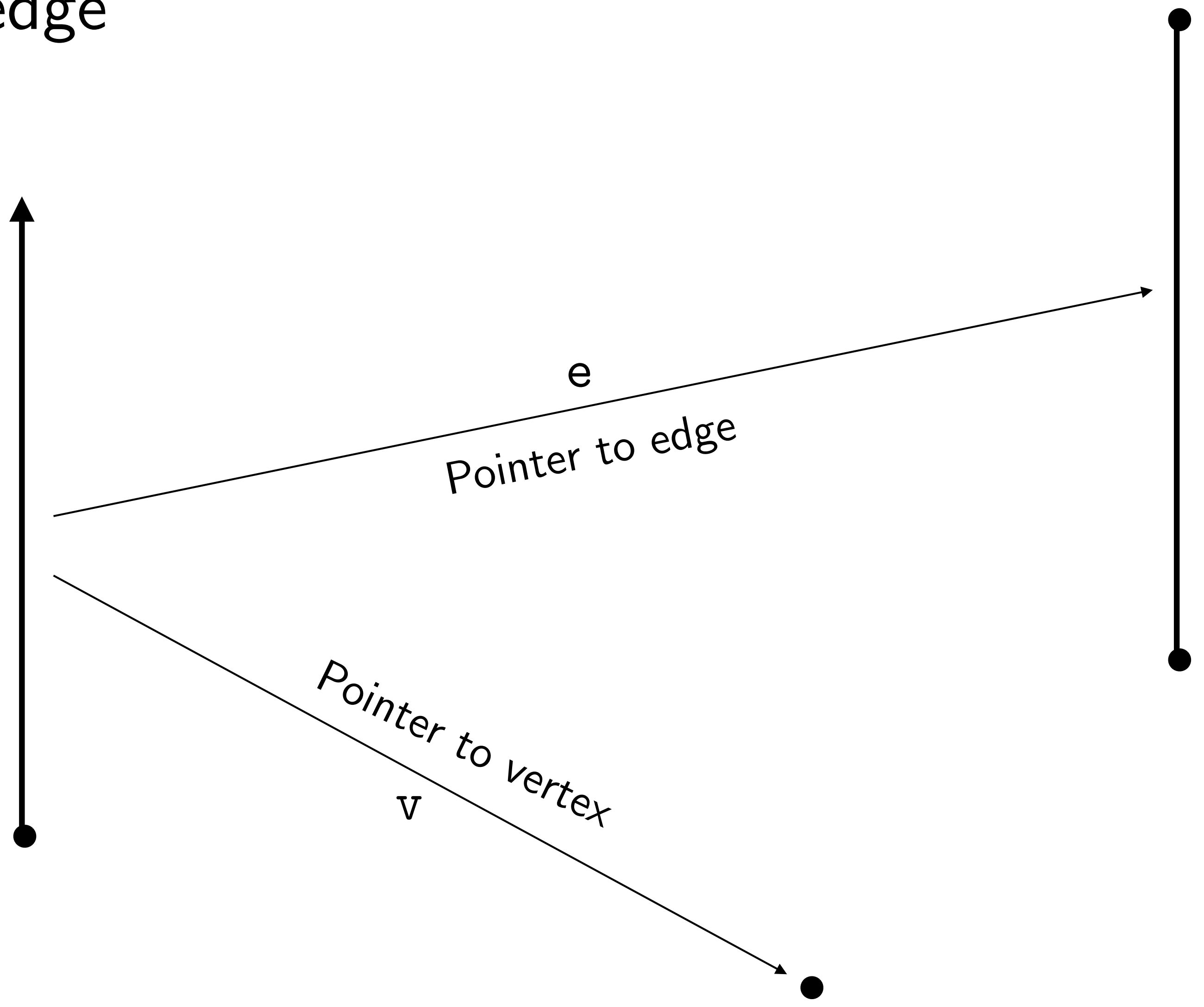

Data Structure

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- Half-edge

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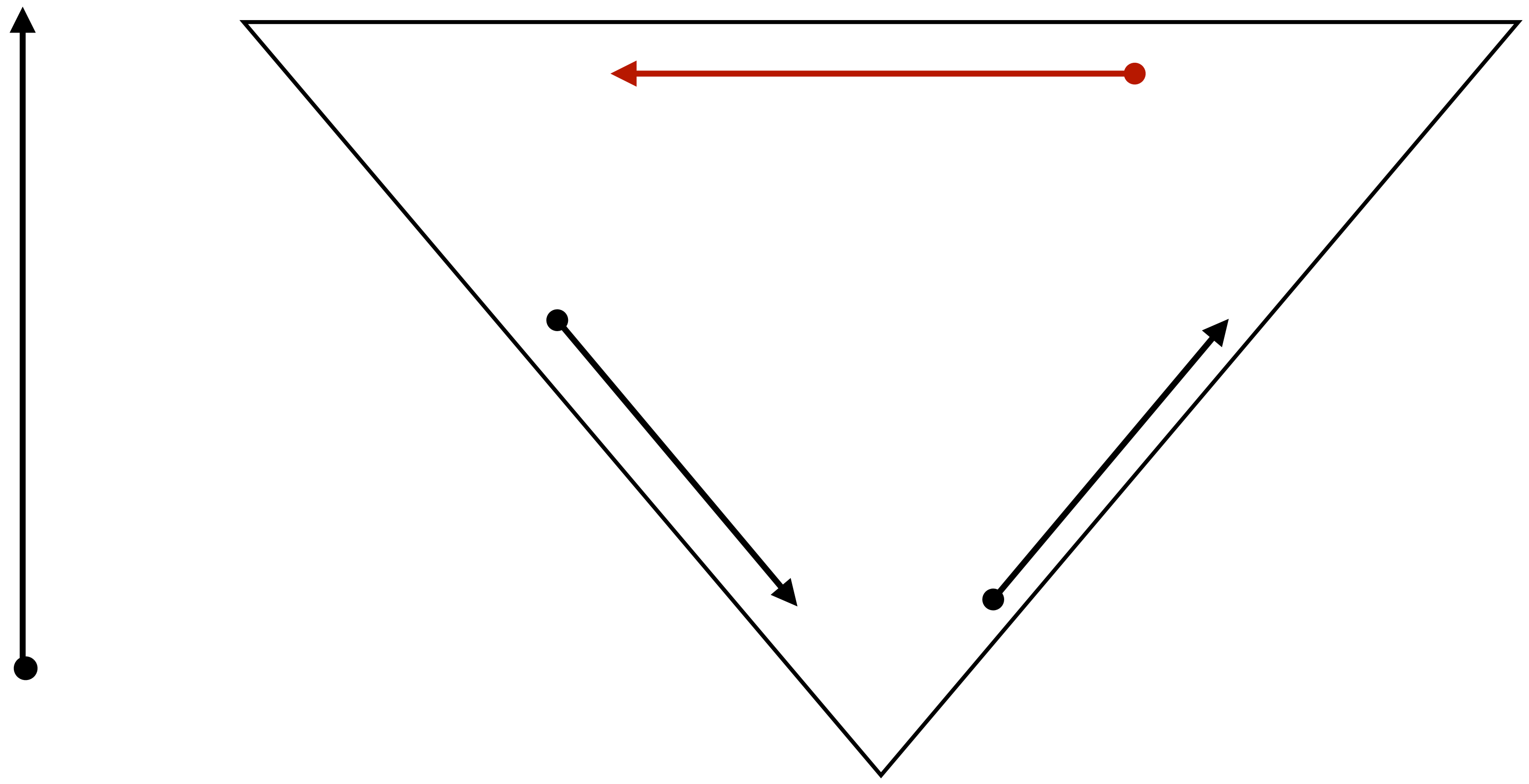
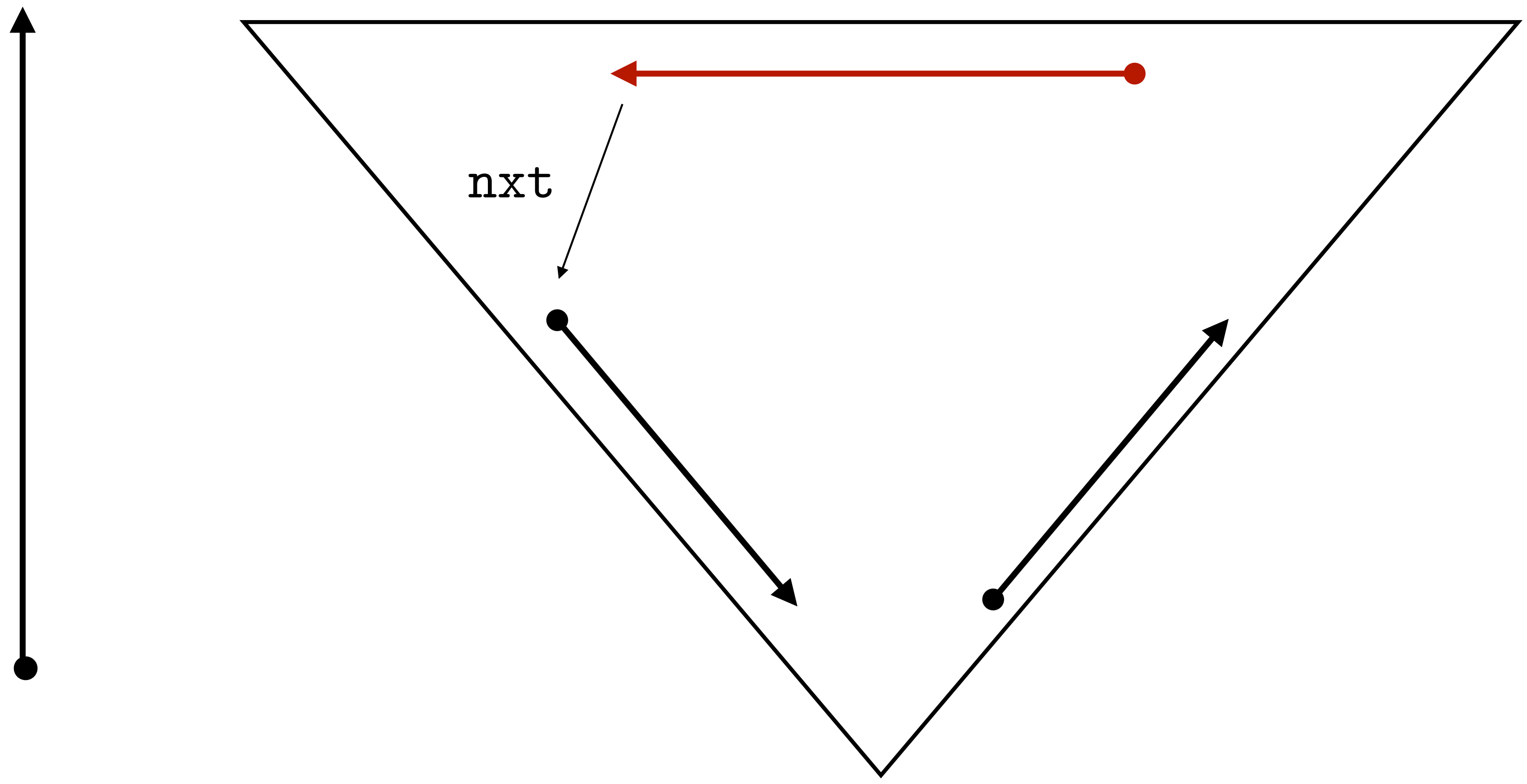


Data Structure

- Half-edge

Pointer to next half-edge



Data Structure

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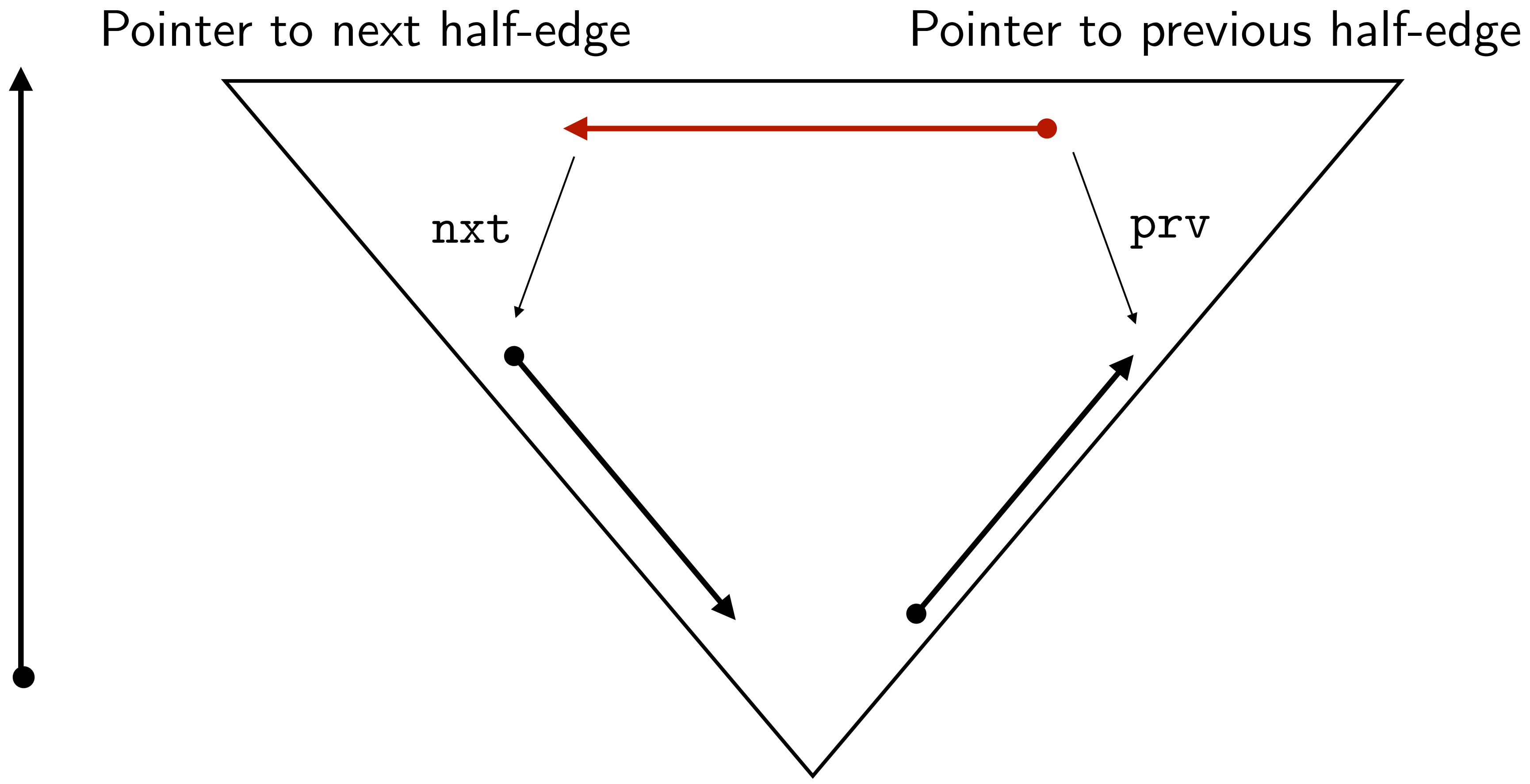

Data Structure

Data Structure

- Vertex
  -

Data Structure

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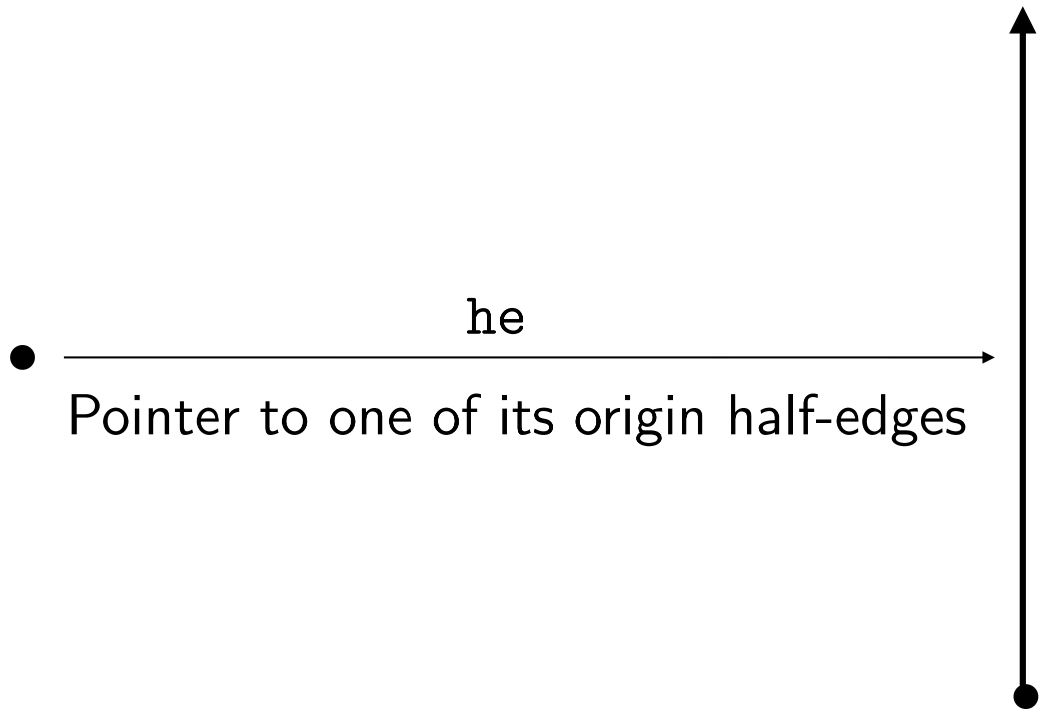
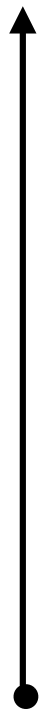


Data Structure

- Vertex

  - $\xrightarrow{\texttt{he}}$

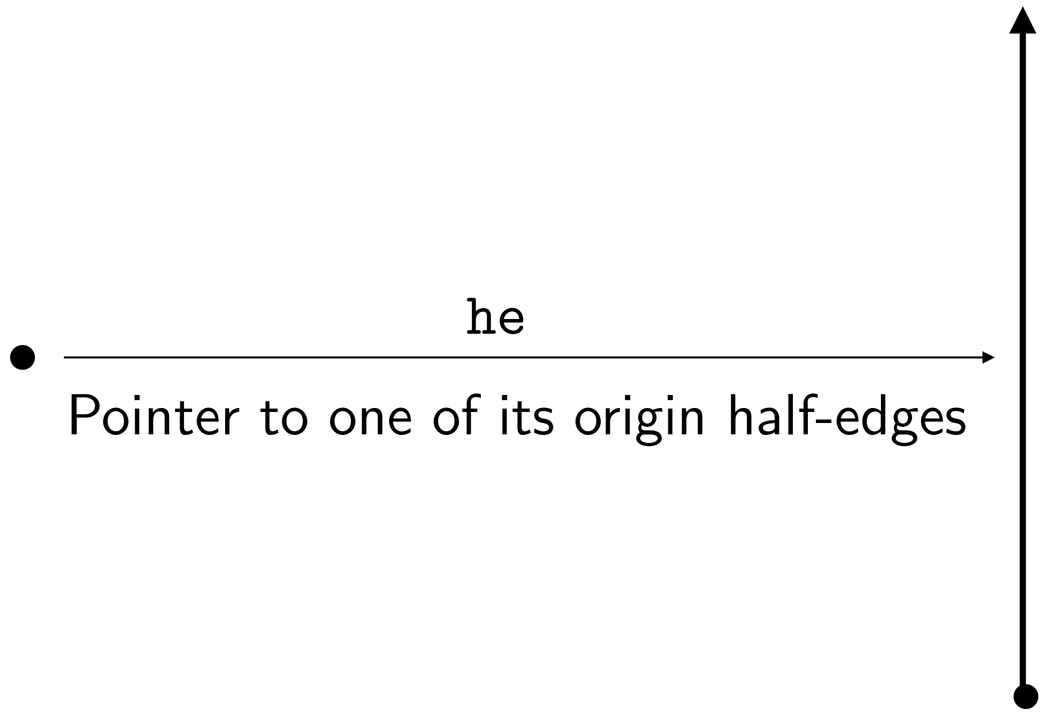
    Pointer to one of its origin half-edges



Any origin half-edge will do!

Data Structure

- Vertex



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Coordinates, (x, y, z), of the vertex

Data Structure

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- Mesh

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  `ltris`: pointer to a list of all triangles

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  `lvrts`: pointer to a list of all vertices

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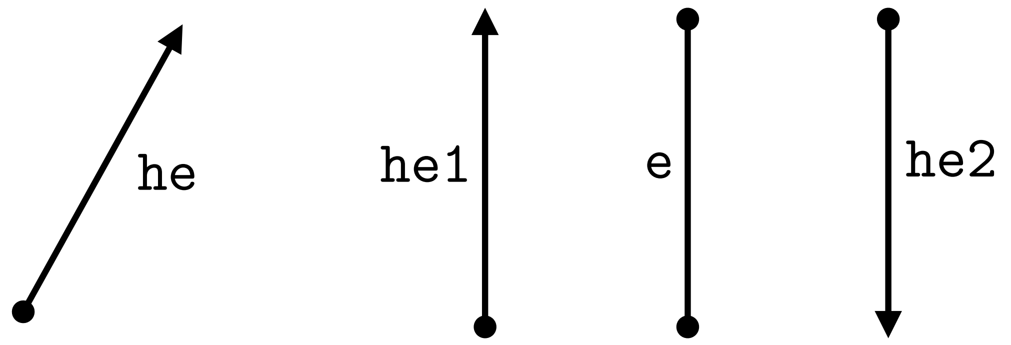

`he = he1 or he = he2?`

Data Structure

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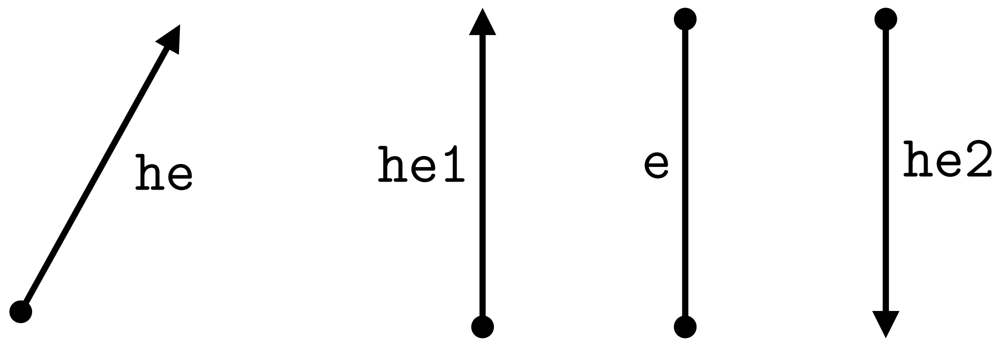

he = he1 or he = he2?

MATE(he)

```
return ( he->e->he1 == he ) ?  he->e->he2 ?  he->e->he1 ;
```

Data Structure

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  Given a pointer, `he`, to a half-edge, return a pointer to *successor* half-edge in a counterclockwise traverse of the cycle of half-edges with the same vertex origin as the half-edge pointed by `he`.

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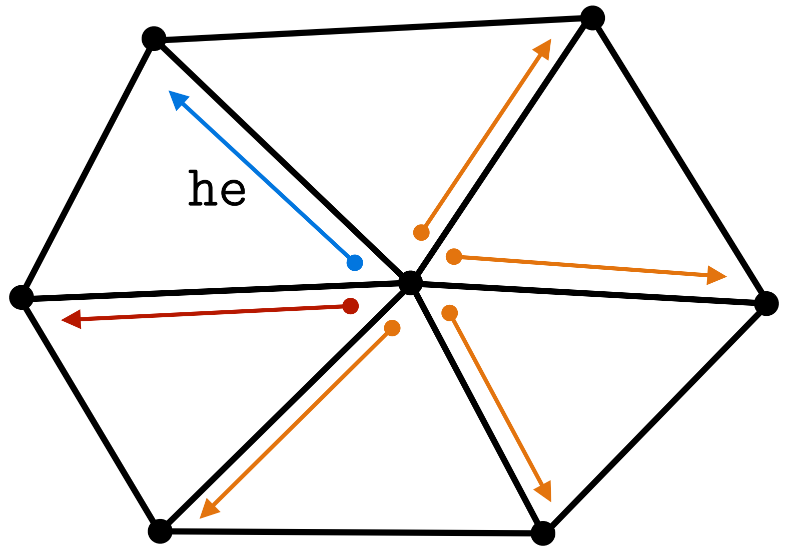

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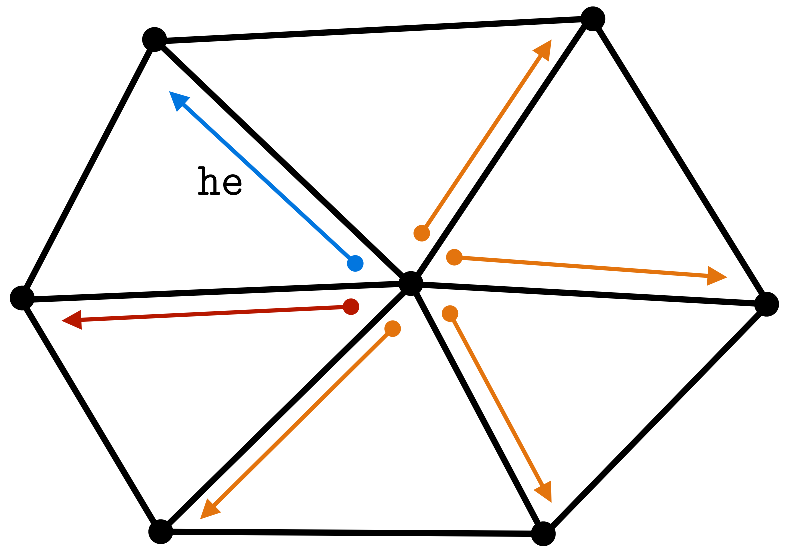

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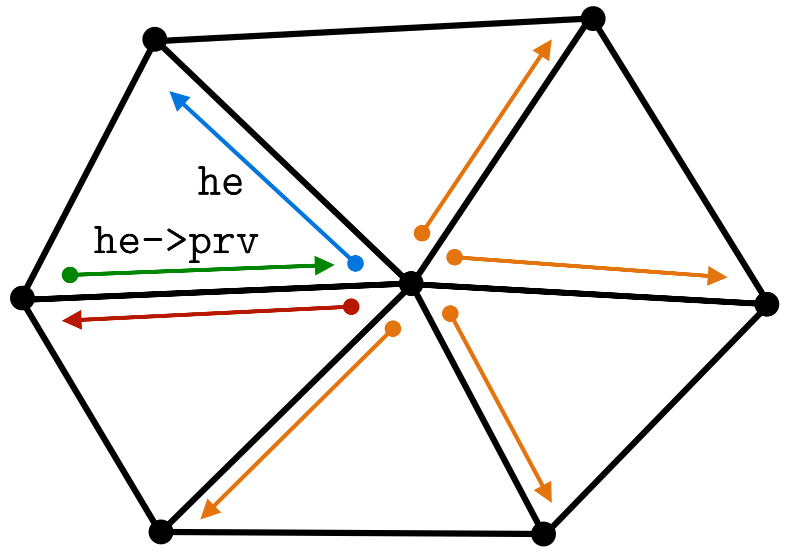

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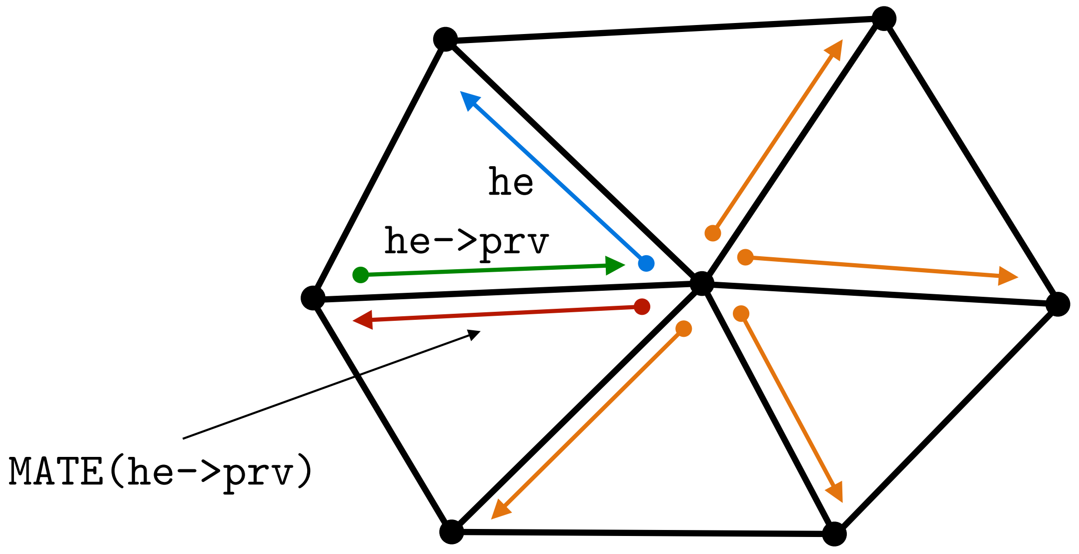

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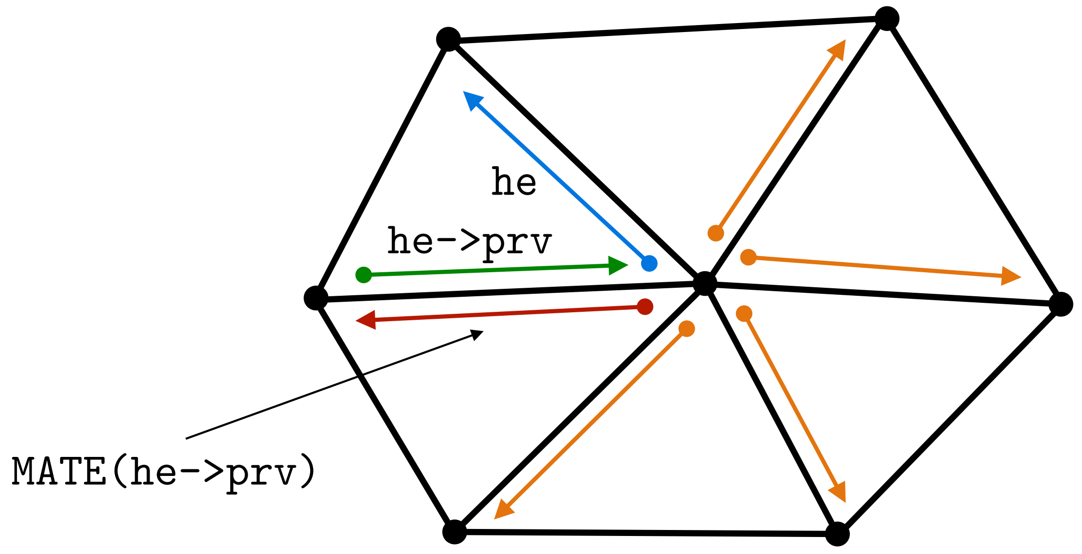

```
SUCC-CC(he)

return MATE(he->prv) ;
```

Data Structure

Data Structure

How can we augment the data structure to store the PPS information?

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A good strategy (*from the software engineering point of view*) is to include a *generic* pointer to *attributes* in every basic element (vertex, half-edge, edge, and triangle) of the data structure.

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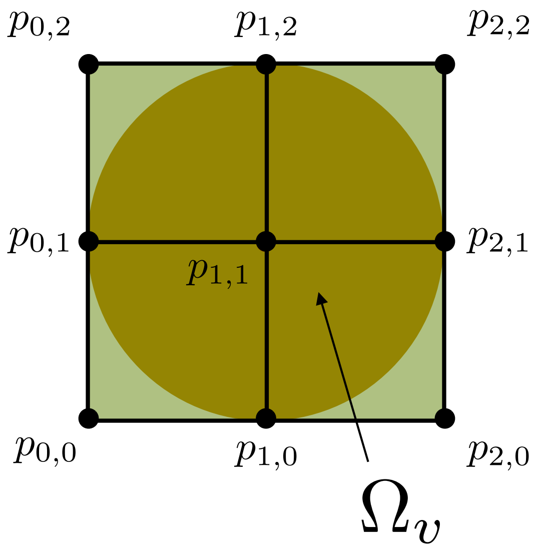

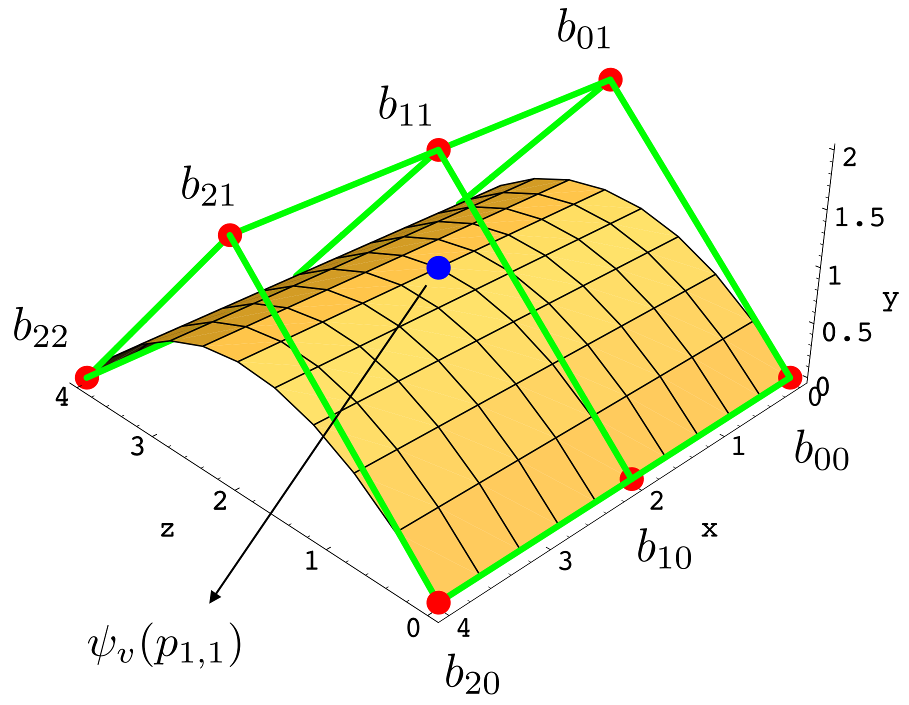

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So, we have *two* vertex attributes: the Bézier patch and the vertex degree.

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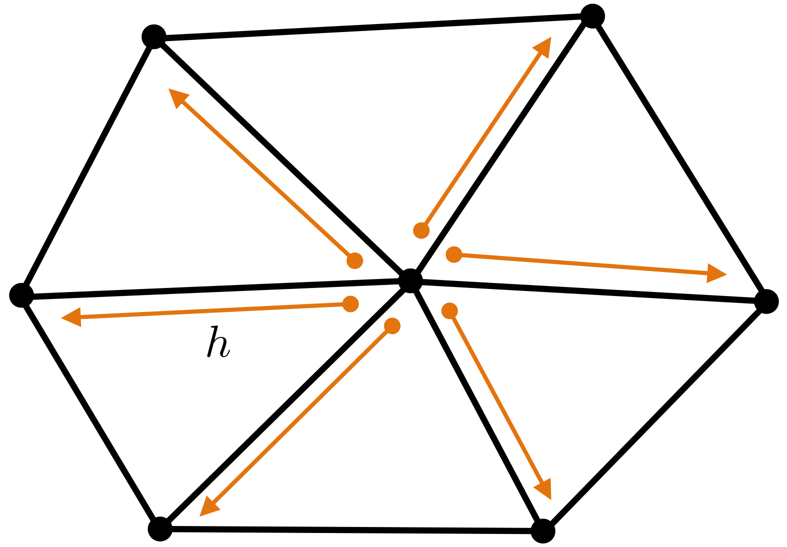

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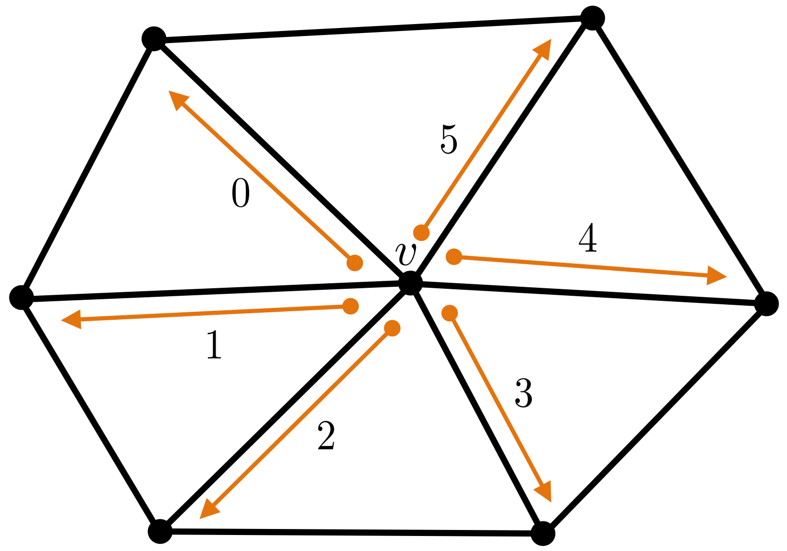

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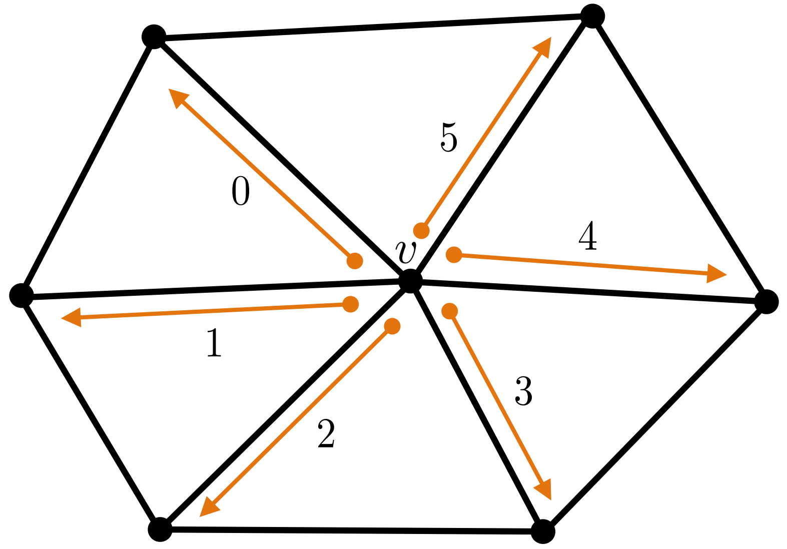

The *id* belongs to $\{0, \ldots, m_v - 1\}$, where m_v is the degree of v.

Data Structure

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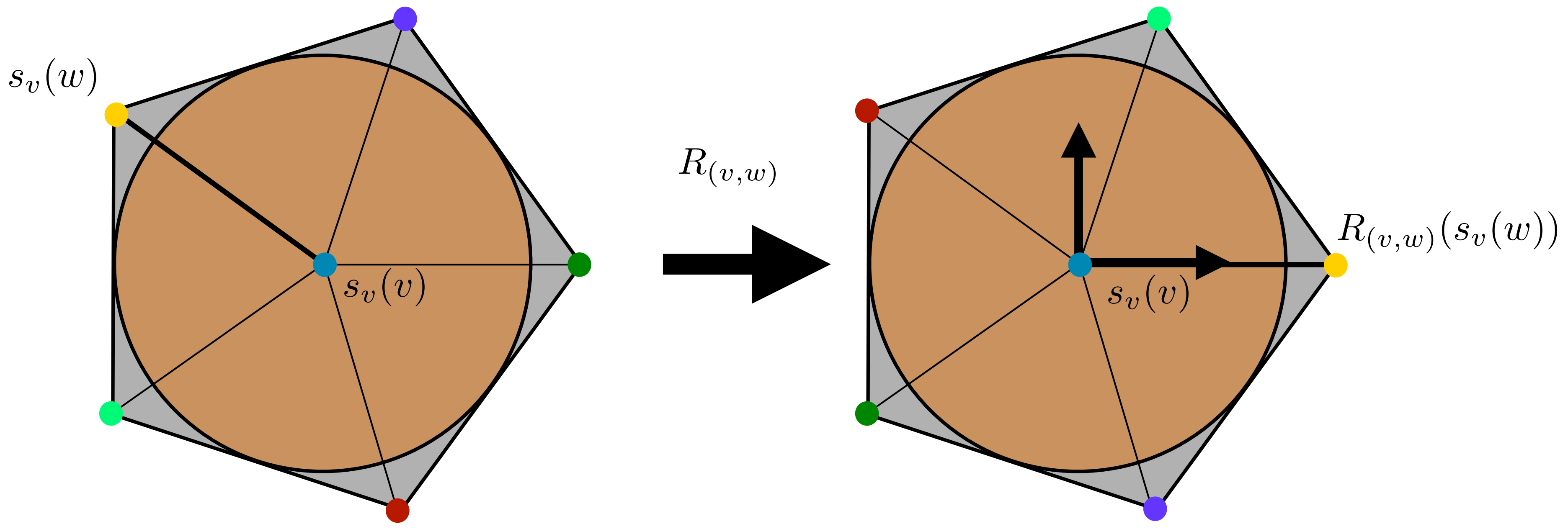

Data Structure

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First, we assume that the half-edge with *id* equals to 0 is the one whose associated edge is mapped to the edge of T_v with endpoints $(2 \cdot l(v), 0)$ and $(2 \cdot l(v) + 1, 0)$ (i.e., the vertices v' and v'_0).

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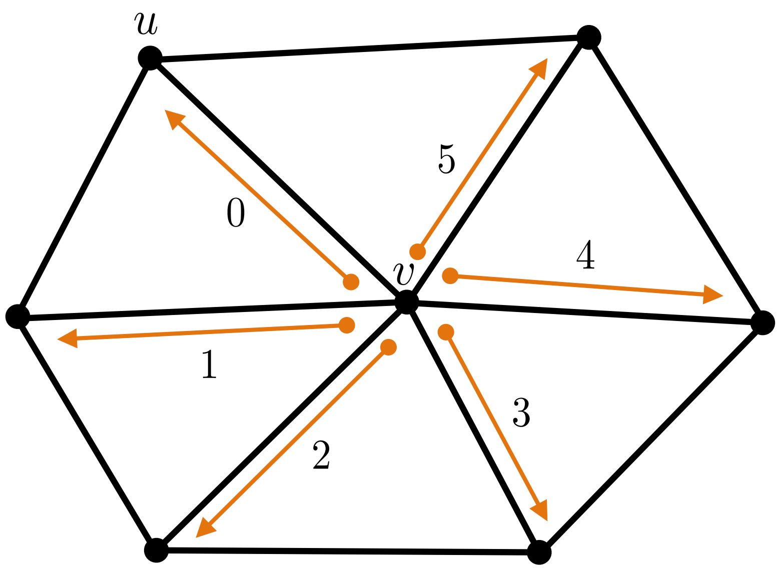

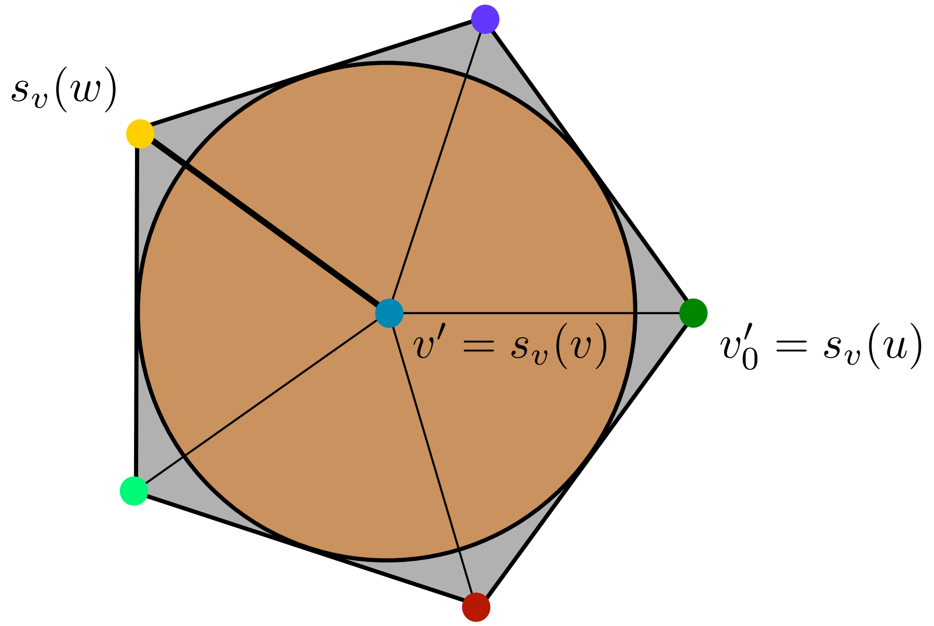

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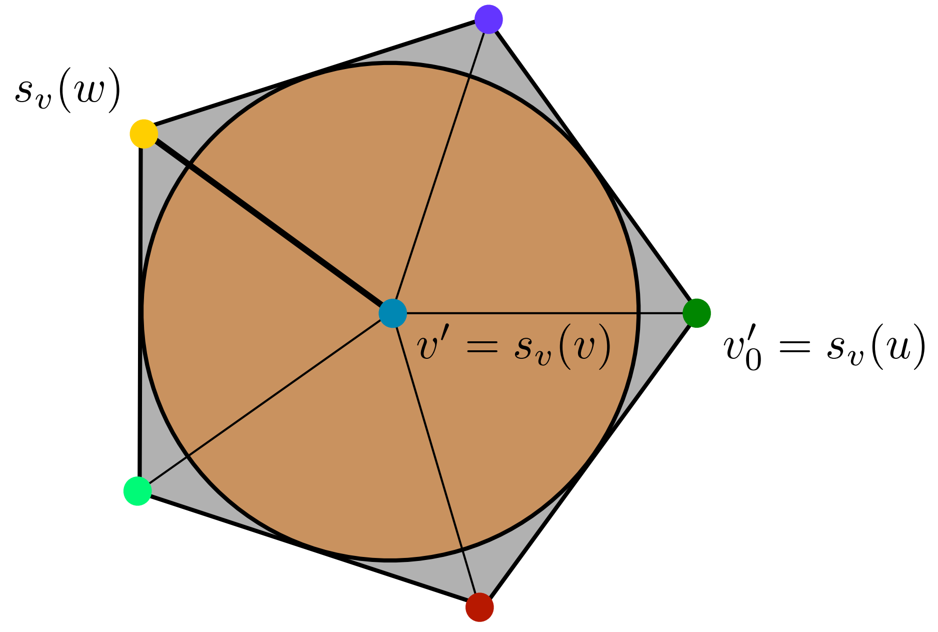

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Likewise, when using subdivision surfaces, some parameters used by Stam's exact evaluation algorithm become triangle attributes.

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Those are abstractions we need to describe the construction only.

Algorithms

Algorithms

How can we compute φ_{uv} at some $p \in \Omega_v$?

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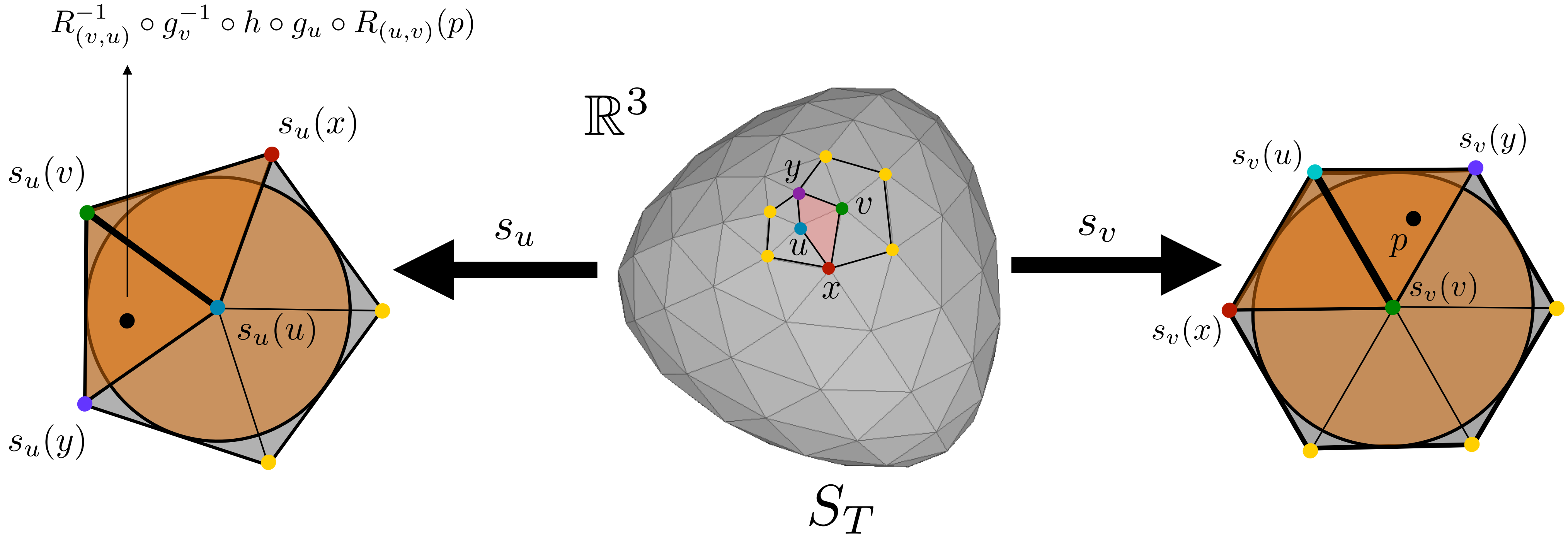

Algorithms

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To start with, let us assume that we have a pointer to the half-edge, h_{vu}, with origin at v and associated with the edge $[u, v]$.

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Recall that

$$\varphi_{uv}(p) = R^{-1}_{(u,v)} \circ g_u^{-1} \circ h \circ g_v \circ R_{(v,u)}(p)\,.$$

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Since we have a pointer to h_{vu}, we can recover its *id* number and then compute the rotation angle of $R_{(v,u)}$, as we saw before.

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So, we are left with g_v, g_u^{-1}, and h, but h is straightforward.

Algorithms

Algorithms

Recall that

$$g_v = \Pi^{-1} \circ f_v \circ \Pi \circ t_v \,,$$

where t_v is the translation that takes $(2 \cdot l(v), 0)$ to $(0, 0)$, Π (resp. Π^{-1}) is the function that converts Cartesian (resp. polar) into polar (Cartesian) coordinates, and f_v is given by

$$f_v(q) = f_v((\theta, r)) = \left(\frac{m_v}{6} \cdot \theta, \frac{\cos(\pi/6)}{\cos(\pi/m_v)} \cdot r \right) \,,$$

where (θ, r) are the polar coordinates of $q \in (-\pi, \pi] \times \mathbb{R}_+$.

Algorithms

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In practice, we assume that every p-domain is centered at $(0,0)$, which makes $g_v = \Pi^{-1} \circ f_v \circ \Pi$. Fine for the sake of implementation!

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Since we have a pointer to h_{vu}, we can access a pointer to v, and thus we can get m_v. Along with the polar coordinates of q (which are immediate to compute), this is all we need to compute g_v.

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Likewise, since we have a pointer to h_{uv}, we can access a pointer to u, and thus we can can get m_u. That's all we need to compute g_u^{-1}.

Algorithms

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Let us turn our attention to the computation of a point on the PPS.

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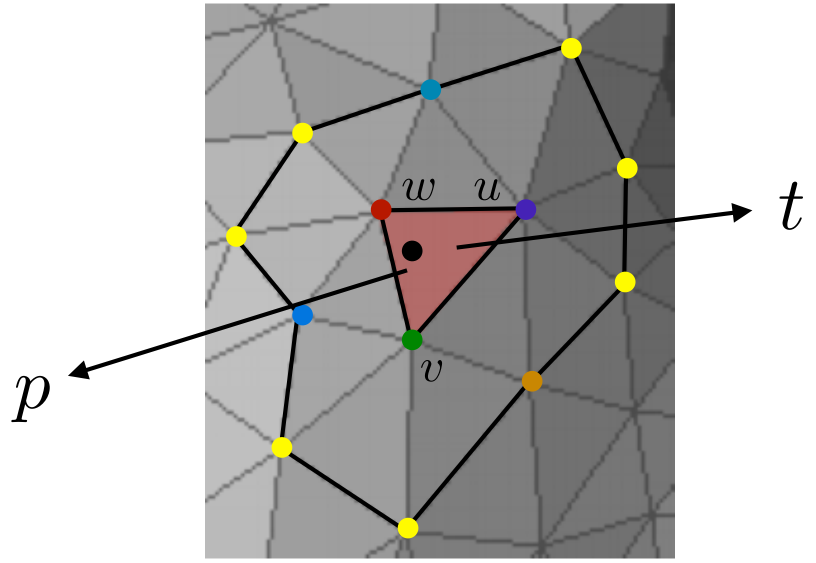
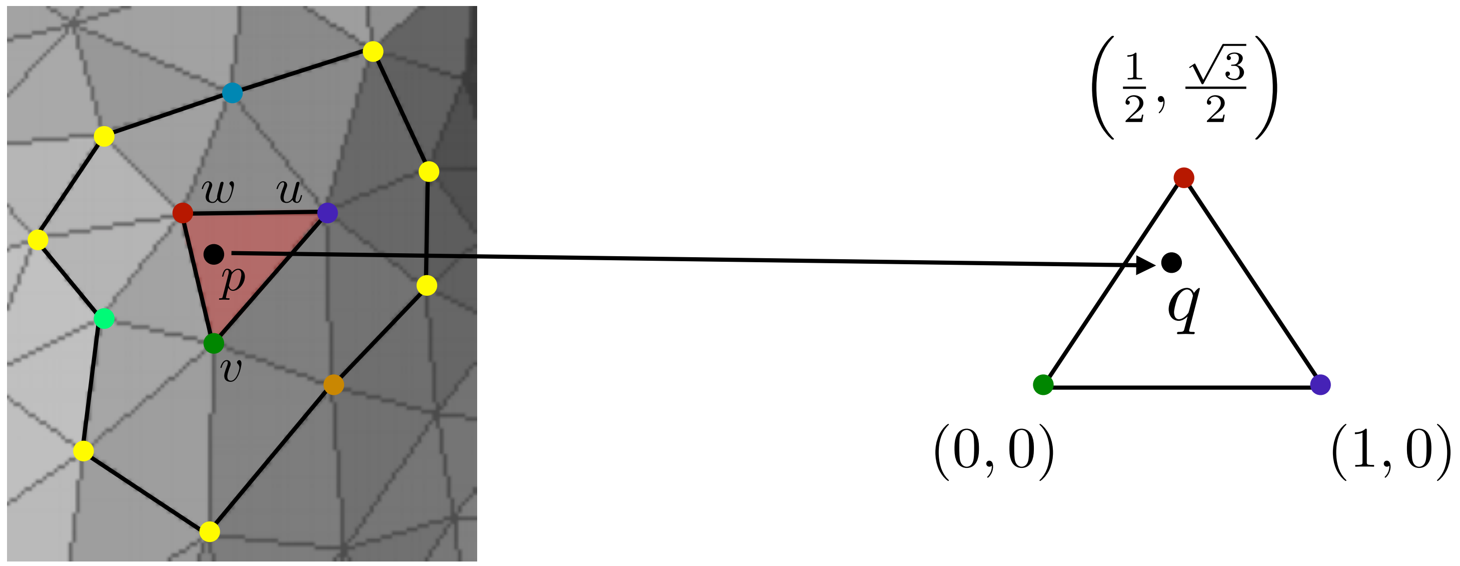


Algorithms

Algorithms

Map p to an equilateral triangle in $\mathbb{R}^2$.



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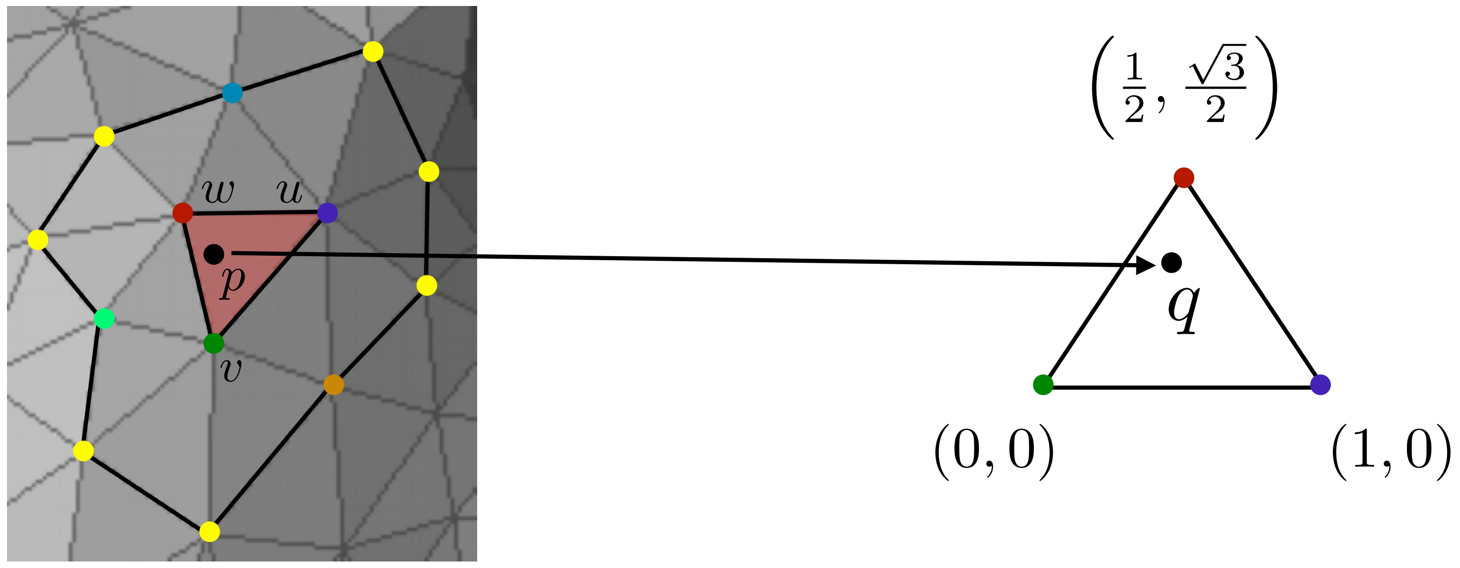

We can do that by using barycentric coordinates.

Algorithms

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Finally, we map q to one of the p-domains: Ω_v, Ω_u, and Ω_w.

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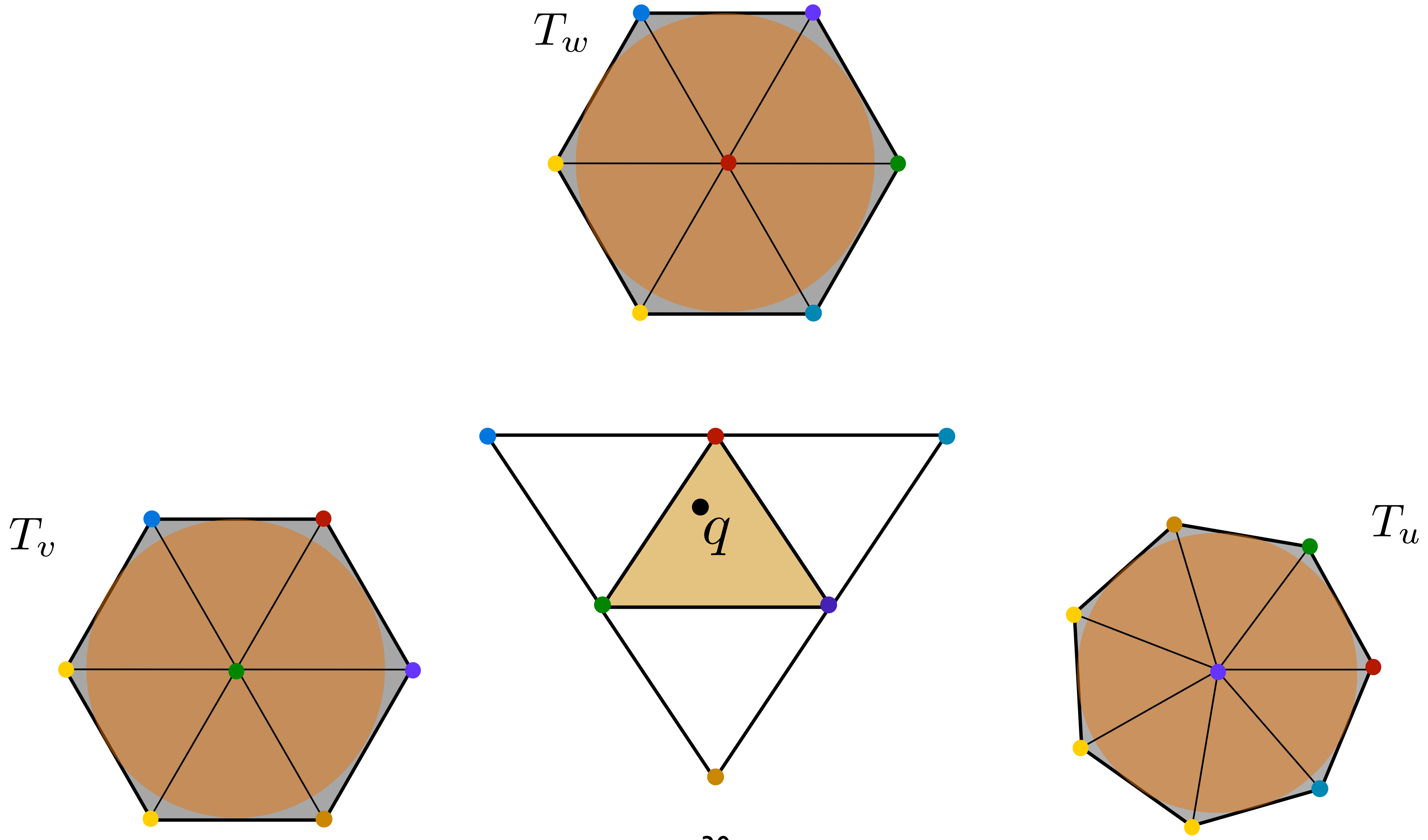

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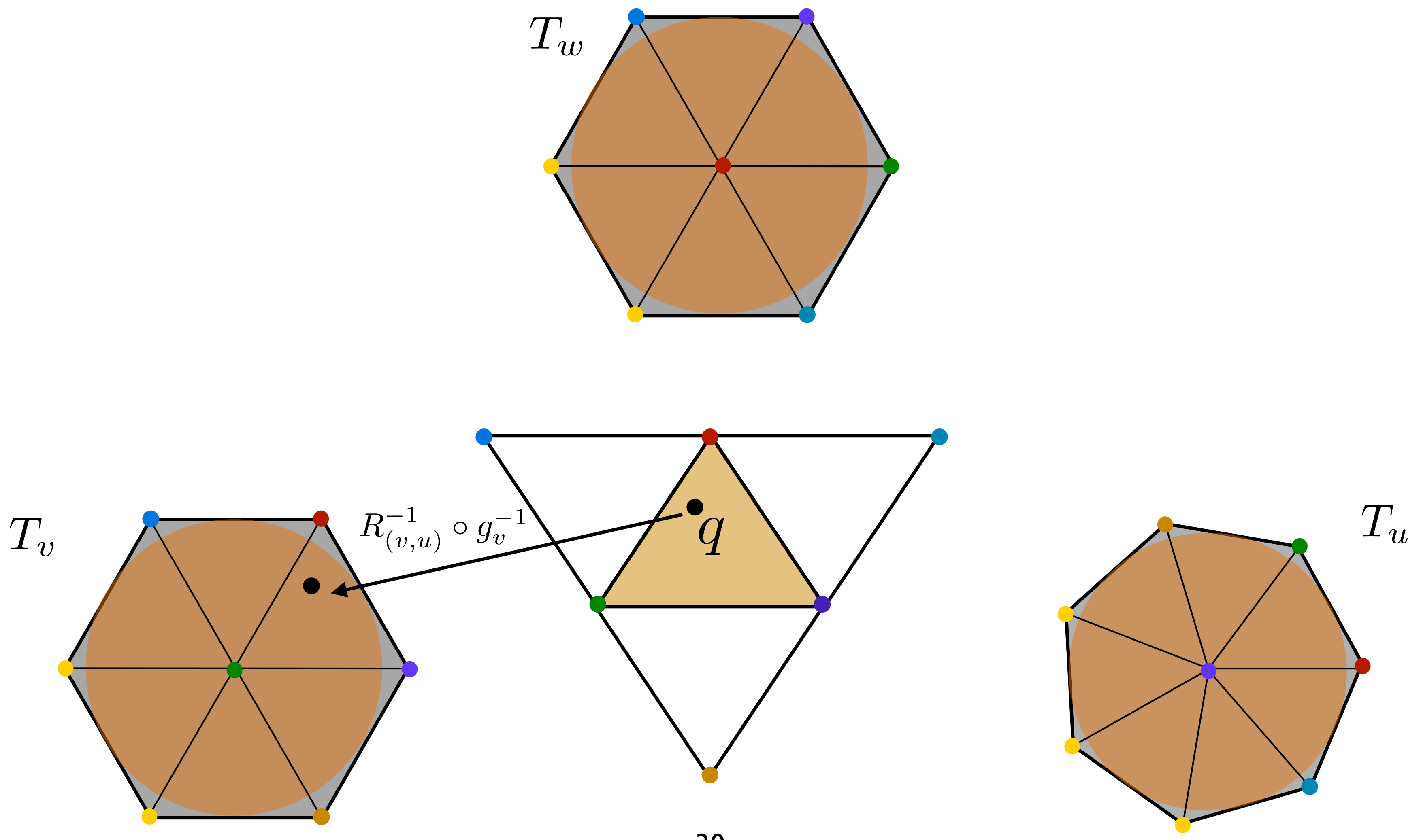

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If that's the case, we choose either u or w and repeat the process.

By construction, we are guaranteed to succeed with one of them!

Algorithms

Algorithms

BE CAREFUL:

Algorithms

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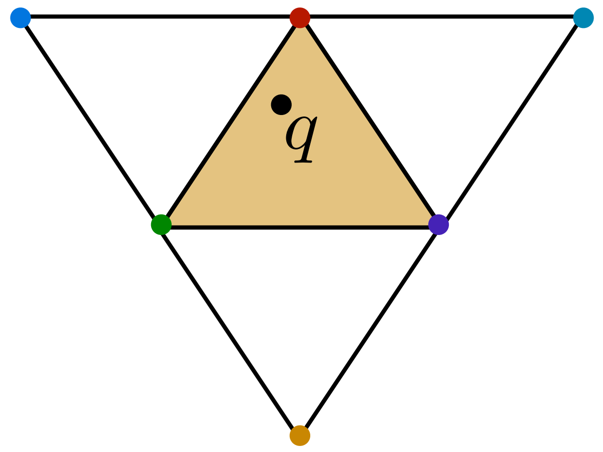
Every time we try a new vertex, the barycentric coordinates of q must change, as the chosen vertex is put in correspondence with the vertex $(0, 0)$ of the canonical triangle (the one colored with "green").



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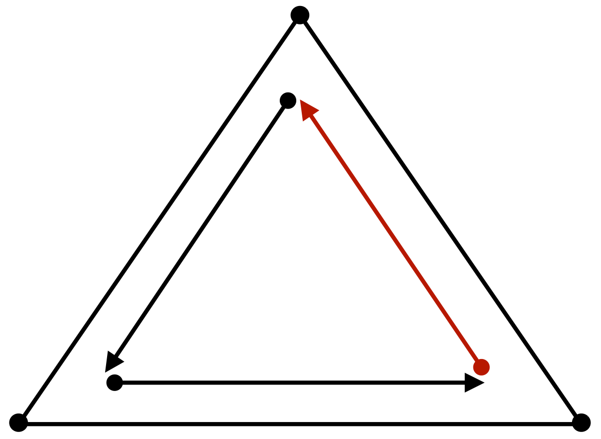

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To compute φ_{uv} and φ_{wv} we need m_u and m_w, which can also be obtained from pointers to the half-edges h_{uw} and h_{wv}, which in turn can be obtained from a pointer to the given triangle t.

Algorithms

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We also need m_v and the half-edge *id* numbers (for the rotation functions), but we can get all that through the half-edge pointers.

Algorithms

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$$\gamma_v(x), \quad \gamma_u(y) \quad \text{and} \quad \gamma_w(z)\,.$$

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Recall that

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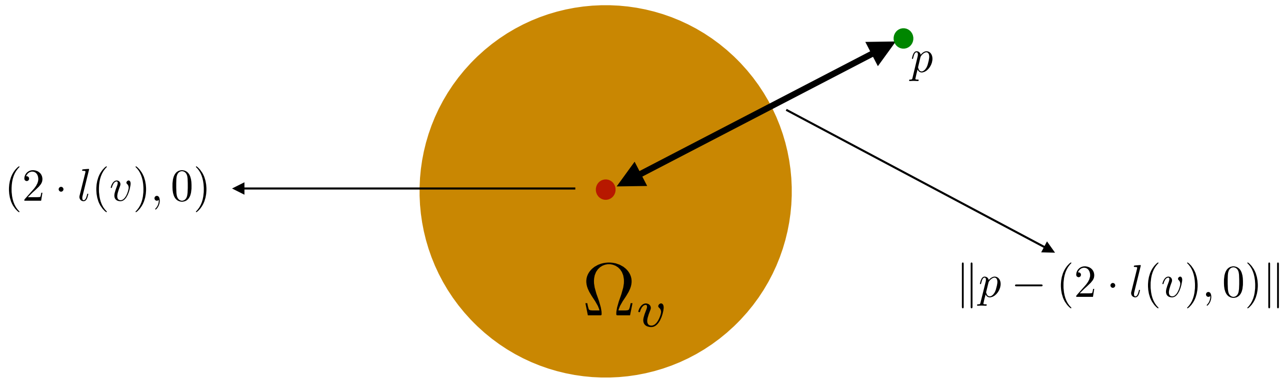

Algorithms

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$\xi : \mathbb{R} \to \mathbb{R}$ is such that, for every $t \in \mathbb{R}$,

$$\xi(t) = \begin{cases} 1 & \text{if } t \leq L_1 \\ 0 & \text{if } t \geq L_2 \\ h(L)/(h(L) + h(1 - L)) & \text{otherwise}\,, \end{cases}$$

where

$$\xi(t) = \begin{cases} 1 & \text{if } t \leq 0 \\ 0 & \text{if } t \geq 1 \\ e^{\frac{2 \cdot e^{-1/t}}{t-1}} & \text{otherwise}\,, \end{cases}$$

L_1, L_2 are constants, with $0 < L_1 < L_2 < 1$, and $L = (t - L_1)/(L_2 - L_1)$.

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Since we let $L_1 = 0.25 \cdot L_2$ and $L_2 = \cos(\pi/m_v)$, we need m_v, which can be obtained through a pointer to v, but such a pointer can be accessed by a pointer to h_{vu} (which we have).

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For γ_u and γ_w, we need m_u and m_w, which we can get in a similar way.

Let $W_v = \gamma_v(x)$, $W_u = \gamma_u(y)$, and $W_w = \gamma_w(z)$.

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If $W_v \neq 0$, compute $\psi_v(x)$ and let $S_v = \psi_v(x) \cdot W_v$. Otherwise, let $S_v = 0$.

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If $W_w \neq 0$, compute $\psi_w(z)$ and let $S_w = \psi_w(z) \cdot W_w$. Otherwise, let $S_w = 0$.

It is true that $W_v + W_u + W_w \neq 0$.

Algorithms

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Finally, the point on the PPS is given by

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What we just did was to compute

$$\theta_v(x) = \theta_u(y) = \theta_w(z)\,.$$

Conclusions

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- Currently, we only have the point evaluation procedure in our API.
- To make the PPS useful for practical applications, this API must be extended to include several fundamental operations, such as computation of derivatives, ray intersection, etc.

Suggested Reading

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Unfortunately, there is no proper documentation of the code other than the inline comments. This must change in the future.

Acknowledgments

Acknowledgments

We've worked with the following people to obtain the results shown here:

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Thank You!