Applications of Wavelets in Graphics

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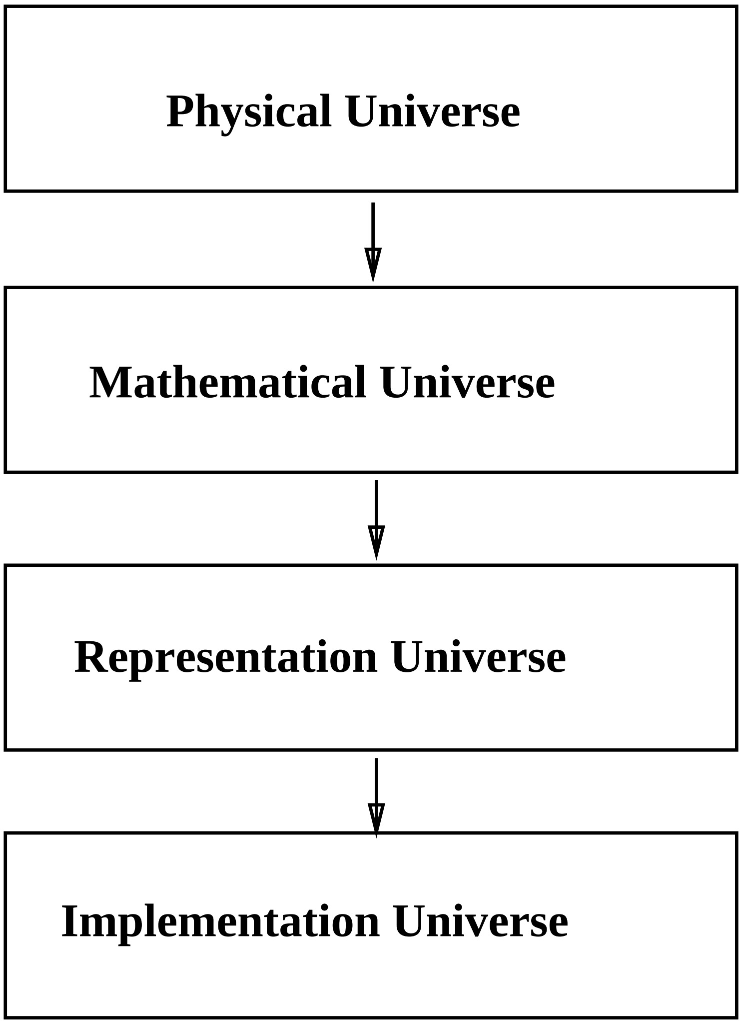
Outline

- Graphical Entities
- Advantages of Wavelets
- Examples of Applications
- Literature in the Area
- Overview of the Course

Graphical Entities

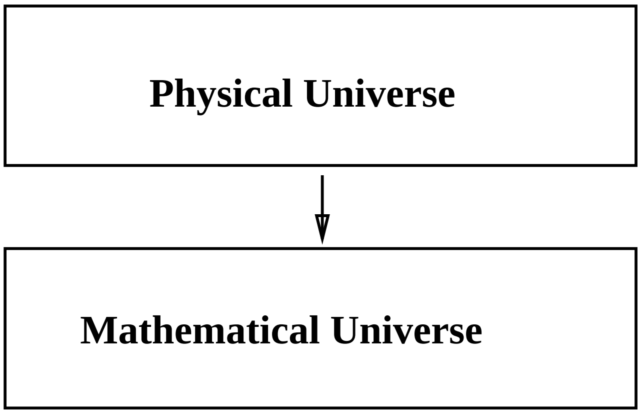
- Graphical Objects
  - Examples:
    - Images
    - Drawings
    - 3D Models
- Operations with Graphical Objects
  - Examples:
    - Filtering
    - Free-Form Editing
    - CSG Ops
- Graphical Problems
  - Examples:
    - Visibility
    - Illumination

Levels of Abstraction



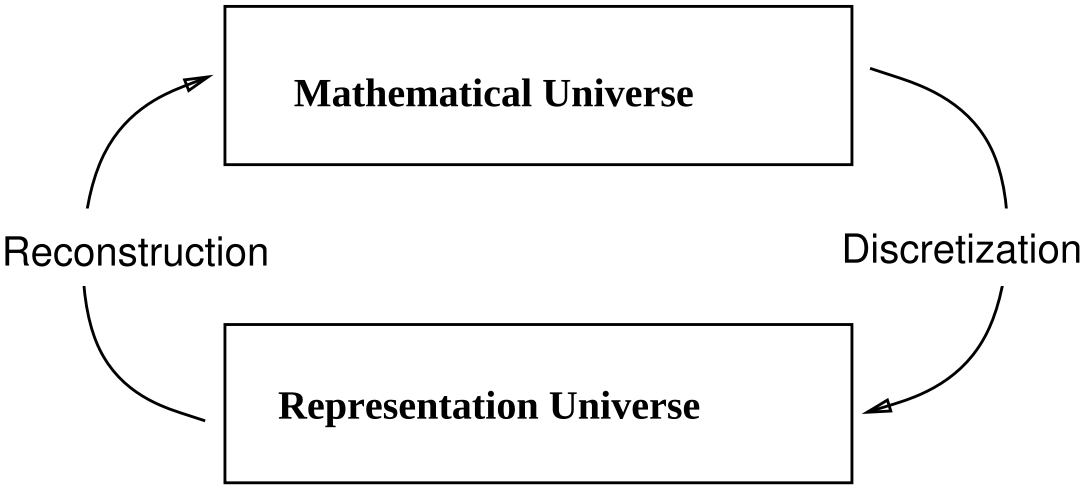
- Continuous $\rightarrow$ Discrete $\rightarrow$ Finite

Wavelets and Models



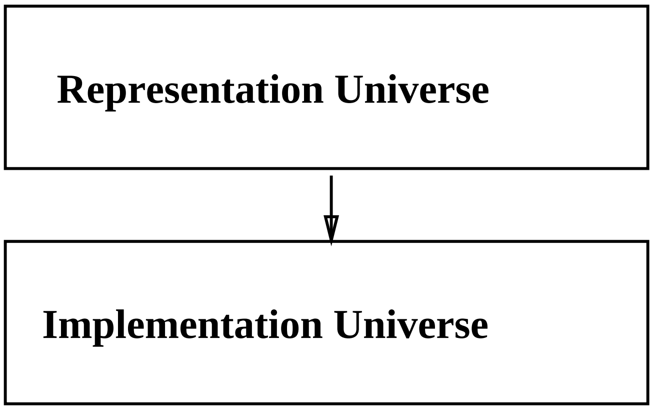
- Rigorous Mathematical Model
  - Function Spaces
    e.g. $L^2(\mathbf{R}^n)$
  - Time and Frequency Localization

Wavelets and Representation



- Effective Representation Scheme
  - Bases, Frames
  - Control over Approximations

Wavelets and Implementation



- Efficient Algorithms
  - Compact Data Structures
  - Recursive Procedures

Wavelets for Graphical Objects

- Graphical Object
  - Shape
  - Attributes
- Wavelet
  - Model for Shape / Attribute Functions
- Properties
  - Capture Relevant Features
  - Adapted Representations
  - Tolerance Control
- Example
  - Parametric Surface with Texture

Wavelets for Graphical Operations

- Equivalent Operations

$$\begin{tikzcd} M \arrow[r, "O_m"] \arrow[d, "D"'] & M' \arrow[d, "D"] \\ R \arrow[r, "O_r"'] & R' \end{tikzcd}$$

* Exact Operations

- Example
  - Low-Pass Filter

Wavelets in Graphical Problems

- Solution of Inverse Problems
  - Functionals
  - Inverse of Operators
- Example
  - Radiosity

Graphics Areas

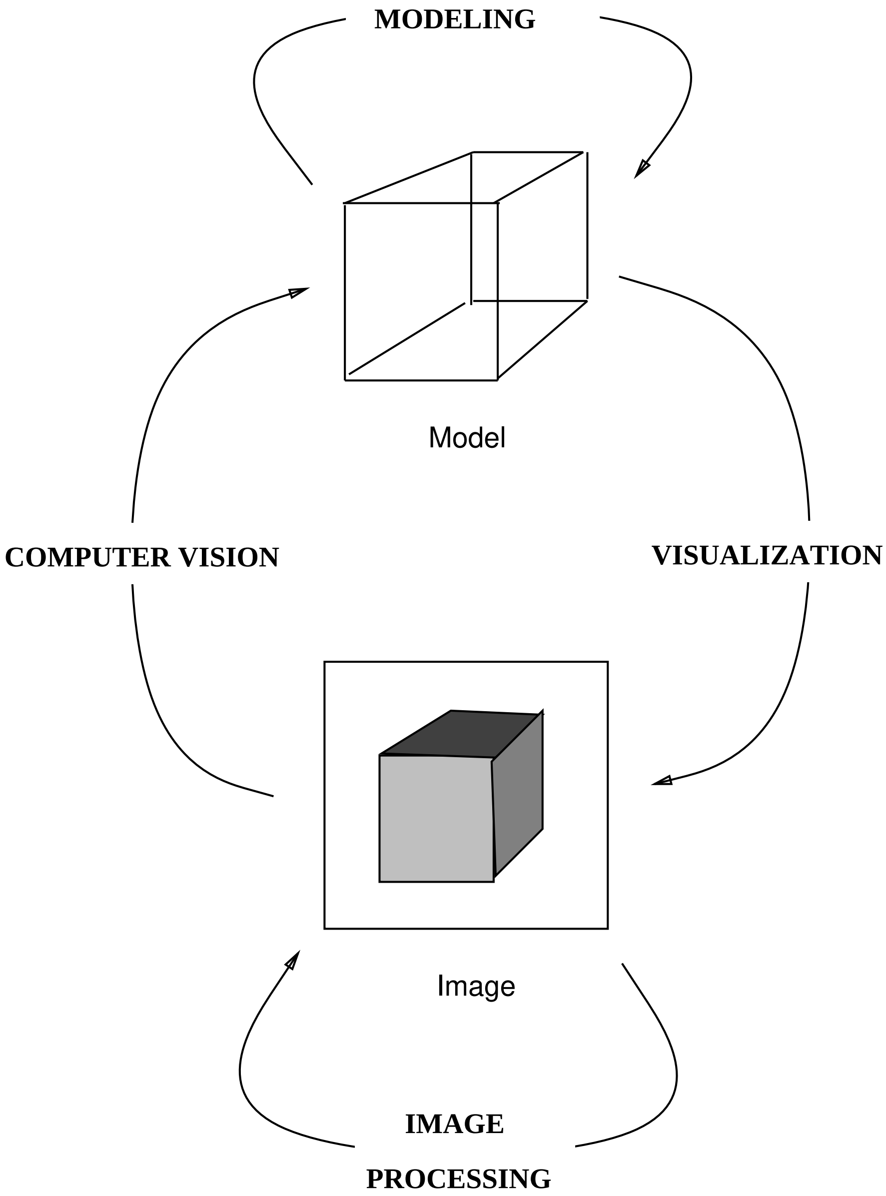


Image Processing

- Images
  - Trivial Geometry
  - Complex Attribute Function
- Wavelets
  - Representation / Operations

    $f : [0,1] \times [0,1] \to \mathbf{R}$
- Advantages
  - Multiscale Model
    - Resolution
  - Compact Representation
    - Approximations
  - Hierarchical Structures
    - Fast Algorithms
- Relation with
  - QMF filters
  - Image Piramids

Applications in Image Processing

- Image Compression
  - **Subband Image Coding with Three-tap Pyramids,**
    Edward H. Adelson, Eero P. Simoncelli,
    Picture Coding Symposium, (1990)
  - **Compressing Still and Moving Images with Wavelets,**
    Hilton,L., Jawerth, D., Sengupta, N.,
    Multimedia Systems (1994)
- Digital Painting
  - **Multiresolution Painting and Compositing,**
    Berman, D., Salesin, D.,
    Siggraph (1994)
  - **B-Spline Wavelet Painting,**
    Perlin, K., Velho,
    L., Siggraph (1994)
  - **Live Paint: Painting with Procedural Multiscale Textures,**
    Perlin, K., Velho, L.,
    Siggraph (1995)

Image Analysis

- Problems:
  - Feature Extraction
  - Model Inference
- Wavelet -¿ Model
  - Multiscale Structures
  - Detection of Singularities
  - Hierarchical Search
- Relation with
  - Fractals
  - Edge Detection

Applications in Image Analysis

- Feature Detection
  - **Corner Detection using Spline Wavelets,**
    Chan, A, Chui, C., Zha, J. Liu, Q.
    SPIE Conference (1991)
- Pattern Matching
  - **Fast Multiresolution Image Querying,**
    Jacobs, C., Filkelstein, A., Salesin, D.
    Siggraph (1995)
- Model Synthesis
  - **Pyramid-Based Texture Analysis/Synthesis,**
    Heeger, D., Bergen, J.
    Siggraph (1995)

Modeling

- Problem:
  - Shape is Complex
  - Attribute Varies
- Wavelet
  - Shape Functions
  - Modeling Operations
- Parametric Surfaces

  $f : U \to \mathbf{R}^3$
  - Arbitrary Topology
- Implicit Surfaces / Volumes

  $g : \mathbf{R}^3 \to \mathbf{R}$
  - Control over Tubular Neighborhood

Applications in Modeling I

- Parametric Models
  - **Multiresolution Curves,**
    Finkelstein,A., Salesin, D.,
    Siggraph, (1994)
  - **Variational Modeling with Wavelets,**
    Gortler,S., Cohen,
    M. (1994)
  - **Multiresolution Analysis of Arbitrary Meshes,**
    Eck, M., De Rose,T.,
    Siggraph (1995)
  - **Spherical Wavelets: Efficiently Representing Functions on the Sphere,**
    Schroder,P., Sweldens, W.,
    Siggraph (1995)
  - **From Global to Local, Shape Models with Fractal Priors,**
    Vemuri, B., Radisavljevic, A,
    TOG (1995)

Applications in Modeling II

- Implicit Models
  - **Surflets,**
    Perlin, K., Zhu, B.,
    Siggraph Course Notes (1993)
  - **Volumetric Wavelet Models,**
    Muraki, S.
    IEEE CG&A (1994)
  - **Multiscale Implicit Models,**
    Velho, L., Terzopoulos, D., Gomes, J.,
    Sibgrapi (1994)
- Conversion
  - **Constructing Implicit Shape Models from Boundary Data,**
    Velho, L., Terzopoulos, D., Gomes, J.,
    GMIP (1995)

Animation

- Motion Parameters
  - Scalar Functions over Time: $\theta(t)$
    - Interrelation
    - Constraints

- Physics-Based Approach
  - Energy Functionals
    $f(\theta) = \int E(\theta) dt$
  - Constrained Minimization
    $\min f(\theta, \theta' \theta'')$ s.t. $c_i(\theta) \leq 0$
  - Iterative Solution

- Wavelets represent Parameter Functions
  - Fast Convergence

- Relation with
  - Multigrid Methods

Applications in Animation

- Motion Synthesis
  - **Hierarchical Spacetime Control,**
    Liu,Z., Gortler,J., Cohen, M.,
    Siggraph, (1994)
- Motion Learning
  - **Quick Tuning of a Reference Locomotion Gait,**
    Nougaret, J. Hegron, G.,
    Computer Animation (1995)

Visualization

- Various Problems
  - Global Illumination -¿ Energy Ballance
- Illumination Equation

  $$\omega \cdot \nabla\phi(r,\omega) + \sigma(r)\phi(r,\omega)$$
  $$= \epsilon(r,\omega) + \int K(r, w' \cdot w)\phi(r,\omega')d\omega'$$
- Fredholm Integral Equation

  $$f = g + Kf$$
  $$f = (I - K)^{-1}g$$
- Wavelet represent Integral Operator
  - Diagonalize K
  - Error Estimation
- Relation with
  - Finite Elements

Applications in Visualization

- Illumination
  - **Wavelet Radiosity,**
    Gortler,S., Schroder,P., Cohen,M., Hanrahan, P.,
    Siggraph, (1993)
  - **Textures and Radiosity: Controlling Emission and Reflection with Texture Maps,**
    Gershbein S., Schroder,P.,
    Siggraph, (1995)
  - **Wavelet Radiance,**
    Christensen, P., Salesin, D., De Rose, T.,
    Eurographics Rendering Workshop, (1996)
  - **Spherical Wavelets: Illumination and Texture Processing,**
    Schroder,P., Sweldens, W.,
    Preprint (1996)

Other Applications

- Morphing
  - **Wavelet-Based Volume Morphing,**
    He, T., Wang, S., Kaufman, A.,
    Visualization (1994)

- Volume Rendering
  - **A Multiresolution Framework for Volume Rendering,**
    Wesermann, R.
    Volume Visualization (1995)
  - **A New Method to Approximate the Volume Rendering Equation using Wavelet Basis,**
    Grob, M., Lippert, L., Koch, R.,
    Volume Visualization (1995)

Conclusions

- Overview

  Recent Area (last 2 years)
  - Image Processing (Mature)
  - Visualization (Consolidating)
  - Modeling (Early Stages)

- General Texts

  - **Wavelets for Computer Graphics,**

    Fournier, A. (organizer)

    Siggraph Course Notes (1994/5)

  - **Wavelets for Computer Graphics: A Primer,**

    Stollnitz, E., De Rose, T., Salesin, D.

    IEEE CG&A (1995)

Course Overview

- Motivation
- Overview of Wavelets in Graphics
- Basic Wavelet Theory
- Computation with Wavelets
- Wavelets in R^n
- Images
- Generalized Wavelets
- Models